NELSON SENIOR MATHS METHODS 12

FULLY WORKED SOLUTIONS

Chapter 7 Logarithmic functions

Exercise 7.01 Indices and logarithms

Concepts and techniques

1 **a** $\log_5(25) = 2 \Rightarrow 25 = 5^2$

b $\log_4(16) = 2 \Rightarrow 16 = 4^2$

c $\log_5(125) = 3 \Rightarrow 125 = 5^3$

d $\log_2(16) = 4 \Rightarrow 16 = 2^4$

e $\log_3(3) = 1 \Rightarrow 3 = 3^1$

f $\log_7(49) = 2 \Rightarrow 49 = 7^2$

g $\log_2(128) = 7 \Rightarrow 128 = 2^7$

h $\log_5(1) = 0 \Rightarrow 1 = 5^0$

2 **a** $\log_8(2) = \frac{1}{3} \Rightarrow 2 = 8^{\frac{1}{3}}$

b $\log_4\left(\frac{1}{2}\right) = -\frac{1}{2} \Rightarrow \frac{1}{2} = 4^{-\frac{1}{2}}$

c $\log_4(\sqrt[4]{7}) = \frac{1}{4} \Rightarrow \sqrt[4]{7} = 7^{\frac{1}{4}}$

d $\log_3\left(\frac{1}{\sqrt[3]{3}}\right) = -\frac{1}{3} \Rightarrow \frac{1}{\sqrt[3]{3}} = 3^{-\frac{1}{3}}$

e $\log_2\left(\frac{\sqrt{2}}{4}\right) = -\frac{3}{2} \Rightarrow \frac{\sqrt{2}}{4} = 2^{-\frac{3}{2}}$

f $\log_a(b) = c \Rightarrow b = a^c$

g $\log_c(\sqrt{a}) = 3m \Rightarrow \sqrt{a} = c^{3m}$

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3 **a** $7^2 = 49 \Rightarrow \log_7(49) = 2$

b $3^3 = 27 \Rightarrow \log_3(27) = 3$

c $2^4 = 16 \Rightarrow \log_2(16) = 4$

d $5^3 = 125 \Rightarrow \log_5(125) = 3$

e $11^0 = 1 \Rightarrow \log_{11}(1) = 0$

f $(2)^0 = 1 \Rightarrow \log_2(1) = 0$

4 **a** $5^{-2} = \frac{1}{25} \Rightarrow \log_5\left(\frac{1}{25}\right) = -2$

b $4^{-2} = \frac{1}{16} \Rightarrow \log_4\left(\frac{1}{16}\right) = -2$

c $10^{-3} = \frac{1}{1000} \Rightarrow \log_{10}\left(\frac{1}{1000}\right) = -3$

d $\left(\frac{1}{4}\right)^4 = \frac{1}{81} \Rightarrow \log_{\frac{1}{3}}\left(\frac{1}{81}\right) = 4$ or $3^{-4} = \frac{1}{81} \Rightarrow \log_3\left(\frac{1}{81}\right) = -4$

e $\left(\frac{1}{4}\right)^3 = \frac{1}{64} \Rightarrow \log_{\frac{1}{4}}\left(\frac{1}{64}\right) = 3$ or $(4^{-3}) = \frac{1}{64} \Rightarrow \log_4\left(\frac{1}{64}\right) = -3$

f $\left(\frac{1}{2}\right)^3 = \frac{1}{8} \Rightarrow \log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$ or $2^{-3} = \frac{1}{8} \Rightarrow \log_2\left(\frac{1}{8}\right) = -3$

g $6^{\frac{2}{3}} = \sqrt[3]{36} \Rightarrow \log_6\left(\sqrt[3]{36}\right) = \frac{2}{3}$

h $7^{\frac{3}{5}} = \sqrt[5]{343} \Rightarrow \log_7\left(\sqrt[5]{343}\right) = \frac{3}{5}$

i $a^k = m \Rightarrow \log_a(m) = k$

j $b^3 = d \Rightarrow \log_b(d) = 3$

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5 **a** $\log_2(64) = 6$

b $\log_9(81) = 2$

c $\log_3(81) = 4$

d $\log_7(343) = 3$

e $\log_6(216) = 3$

f $\log_5(1) = 0$

g $\log_3(3) = 1$

h $\log_{10}(100\,000) = 5$

i $\log_3(243) = 5$

j $\log_4(1024) = 5$

6 **a** $\log_{\frac{1}{2}}\left(\frac{1}{16}\right) = 4$

b $\log_5\left(\frac{1}{125}\right) = -3$

c $\log_{\frac{1}{4}}(16) = -2$

d $\log_{\frac{1}{4}}\left(\frac{1}{256}\right) = 4$

e $\log_2\left(\frac{1}{128}\right) = -7$

f $\log_{\frac{1}{2}}(512) = -9$

g $\log_{\frac{1}{3}}\left(\frac{1}{81}\right) = 4$

h $\log_7\left(\frac{1}{7}\right) = -1$

i $\log_{\frac{1}{3}}(27) = -3$

j $\log_{\frac{1}{10}}(0.001) = 3$

Reasoning and communication

7 **a** $\log_2\left(\sqrt{2}\right) = \frac{1}{2}$

b $\log_9(9\sqrt{9}) = \frac{3}{2}$

c $\log_4(\sqrt{64}) = \frac{3}{2}$

d $\log_7(\sqrt{343}) = \frac{3}{2}$

e $\log_6(\sqrt[3]{36}) = \frac{2}{3}$

8 **a** Let $\log_4(8) = x$

$\therefore 8 = 4^x$

$2^3 = 2^{2x}$

$x = 1.5$

$\log_4(8) = 1.5$

b Let $\log_9(27) = x$

$\therefore 27 = 9^x$

$3^3 = 3^{2x}$

$x = 1.5$

$\log_9(27) = 1.5$

c Let $\log_8(16) = x$

$\therefore 16 = 8^x$

$2^4 = 2^{3x}$

$x = \frac{4}{3}$

$\log_8(16) = \frac{4}{3}$

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d Let $\log_{64}(32) = x$

$\therefore\ 32 = 64^x$

$2^5 = 2^{6x}$

$x = \dfrac{5}{6}$

$\log_{64}(32) = \dfrac{5}{6}$

e Let $\log_{27}(243) = x$

$\therefore\ 243 = 27^x$

$3^5 = 3^{3x}$

$x = \dfrac{5}{3}$

$\log_{27}(243) = \dfrac{5}{3}$

9 Given $x = \sqrt[m]{a^p}$ show that $\log_a(x^m) = p$

$x = \sqrt[m]{a^p}$

$x = a^{\frac{p}{m}}$

$\therefore\ \log_a(x) = \dfrac{p}{m}$

$\therefore\ m\log_a(x) = p$

$\therefore\ \log_a(x^m) = p$

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10 Given $a = \frac{1}{\sqrt[y]{b^p}}$ show that $\log_b(a^{-y}) = p$

$$a = \frac{1}{\sqrt[y]{b^p}}$$
$$= \frac{1}{b^{\frac{p}{y}}}$$
$$= b^{-\frac{p}{y}}$$
$$\therefore \log_b(a) = -\frac{p}{y}$$
$$\therefore -y\log_b(a) = p$$
$$\therefore \log_b(a^{-y}) = p$$

11 Given $y = \sqrt[3a]{p^m}$, show that $\log_p(y^{3a}) = m$

$$y = \sqrt[3a]{p^m}$$
$$y = p^{\frac{m}{3a}}$$
$$\therefore \log_p(y) = \frac{m}{3a}$$
$$\therefore 3a\log_p(y) = m$$
$$\therefore \log_p(y^{3a}) = m$$

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Exercise 7.02 Properties of logarithms

Concepts and techniques

1 **a** $\log_3(1) = 0$

b $\log_2(1) = 0$

c $2\log_5(1) = 0$

d $\log_x(1) = 0$ for $x > 0$

e $[\log_3(1)]^2 = 0^2 = 0$

f $\log_3(3) = 1$

g $\log_2(2) = 1$

h $2\log_5(5) = 2 \times 1 = 2$

i $\log_x(x) = 1$ for $x > 0$

j $5\log_a(a) = 5 \times 1$ for $a > 0$

2 **a** $\log_7(0)$ is not defined

b $\log_2(0)$ is not defined

c $2\log_5(0)$ is not defined

d $\log_y(0)$ is not defined

e $\log_7(-1)$ is not defined

f $\log_6(-7)$ is not defined

g $\log_4(-x)$ is not defined

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3 **a** $\log_2(64) = 6$

b $\log_4(64) = 3$

c $\log_3\left(\sqrt{3}\right) = 0.5$

d $\log_5\left(5\sqrt{5}\right) = \log_5(5)\ \log_5\left(\sqrt{5}\right) = 1 + 0.5 = 1.5$

e $\log_7\left(\frac{1}{\sqrt{7}}\right) = –0.5$

f $4\log_a\left(\sqrt{a}\right) = 4 \times 0.5 = 2$

g $2\log_a(a^3) = 2 \times 3 = 6$

4 **a** $\log_4(10) + \log_4(2) – \log_4(5) = \log_4(10 \times 2 \div 5) = \log_4(4) = 1$

b $\log_5(25) + \log_5(125) – \log_5(625) = \log_5(25 \times 125 \div 625) = \log_5(5) = 1$

c $\log_{27}(\tfrac{1}{9}) + \log_8(4) = -\frac{2}{3} + \frac{2}{3} = 0$

d $\log_2(16) + \log_2(4) + \log_2(8) = \log_2(16 \times 4 \times 8) = \log_2(2^9) = 9\log_2(2) = 9$

e $\log_4(40) – \log_4(10) – \log_4(4) = \log_4(40 \div 10 \div 4) = \log_4(1) = 0$

f $\log_5(8) – \log_5(4) + 2 = \log_5(8 \div 4) + 2\log_5(5) = \log_5(2 \times 25) = \log_5(50)$

g $\log_8(2) - \log_8\left(\frac{1}{4}\right) = \log_8(2) - \log_8\left(4^{-1}\right) = \log_8(2) + \log_8(4) = \log_8(8) = 1$

h $\log_6(125) – \log_6(32) - \log_6\left(\frac{2}{5}\right)$

$$= \log_6\left(125 \div 32 \div \frac{2}{5}\right) = \log_6\left(\frac{125}{32} \times \frac{5}{2}\right) = \log_6\left(\frac{625}{64}\right)$$

5 **a** $\frac{\log_6(16)}{\log_6(2)} = \log_2(16) = 4$

b $\frac{\log_2(81)}{\log_2(27)} = \log_{27}(81) = x \Rightarrow 81 = 27^x \Rightarrow 3^4 = 3^3 \Rightarrow x = \frac{4}{3} \Rightarrow \frac{\log_2(81)}{\log_2(27)} = \frac{4}{3}$

c $\frac{\log_3(81)}{\log_3(\frac{1}{3})} = \log_{\frac{1}{3}}(81) = –4$

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d $\dfrac{\log_7(2)}{\log_7(0.25)} = \log_{0.25}(2) = x \Rightarrow 2 = 0.25^x \Rightarrow 2 = 2^{-2x} \Rightarrow x = -\dfrac{1}{2}$

$\dfrac{\log_7(2)}{\log_7(0.25)} = -\dfrac{1}{2}$

6 **a** $5\log_4(x) + \log_4\left(x^2\right) - \log_4\left(x^3\right) = \log_4\left(\dfrac{x^5 \times x^2}{x^3}\right) = \log_4(x^4)$

b $3\log_7(x) - 5\log_7(x) + 4\log_7(x) = \log_7\left(\dfrac{x^3 \times x^4}{x^5}\right) = \log_7\left(x^2\right)$

c $4\log_6(x) - \log_6\left(x^2\right) - \log_6\left(x^3\right) = \log_6\left(\dfrac{x^4}{x^2 \times x^3}\right) = \log_6(x^{-1}) = \log_6\left(\dfrac{1}{x}\right)$

d $\log_2(x+2) + \log_2(x+2)^2 = \log_2(x+2)^3$

e $\log_4[(x-1)^3] - \log_4[(x-1)^2] = \log_4\left[\dfrac{(x-1)^3}{(x-1)^2}\right] = \log_4(x-1)$

f $\log_3(x-3) + \log_3(x+3) - \log_3(x^2-9)$

$$= \log_3\left[\frac{(x-3)(x+3)}{(x^2-9)}\right] = \log_3\left[\frac{(x^2-9)}{(x^2-9)}\right] = \log_3(1) = 0$$

7 **a** $\log\left(\dfrac{12a}{10}\right) = \log_{10}\left(\dfrac{12a}{10}\right) = \log(12) + \log(a) - 1$

b $\log_6\left[\left(\dfrac{a}{b}\right)^5\right] = 5\log_6\left(\dfrac{a}{b}\right) = 5\log_6(a) - 5\log_6(b)$

c $\log_3\left(\sqrt[5]{10x^3}\right) = \dfrac{1}{5}\log_3\left(10x^3\right) = \dfrac{1}{5}\log_3(10) + \dfrac{1}{5}\log_3\left(x^3\right) = \dfrac{1}{5}\log_3(10) + \dfrac{3}{5}\log_3(x)$

d $\log_4\left(\dfrac{\sqrt[3]{x^2a}}{y^2}\right) = \log_4\left(x^2a\right)^{\frac{1}{3}} - \log_4(y^2) = \dfrac{2}{3}\log_4(x) + \dfrac{1}{3}\log_4(a) - 2\log_4(y)$

8 **a** $\log_6\left(\sqrt[4]{3}\right) = \dfrac{1}{4}\log_6(3) = \dfrac{1}{4}(0.613) = 0.153$

b $\log_6(2) = \log_6\left(\dfrac{6}{3}\right) = \log_6(6) - \log_6(3) = 1 - 0.613 = 0.387$

c $\log_6(108) = \log_6(36 \times 3) = 2\log_6(6) + \log_6(3) = 2 + 0.613 = 2.613$

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Reasoning and communication

9 Given that $\log_p(7) + \log_p(k) = 0$, find k.

$\log_p(7) = -\log_p(k)$

$-\log_p(7) = \log_p(k)$

$\log_p\left(\frac{1}{7}\right) = \log_p(k)$ as $7^{-1} = \frac{1}{7}$

$k = \frac{1}{7}$

10 Given $\log_a(x) = 4$ and $\log_a(y) = 5$,

a $\log_a(x^2y) = 2\log_a(x) + \log_a(y) = 2 \times 4 + 5 = 13$

b $\log_a(axy) = \log_a(a) + \log_a(x) + \log_a(y) = 1 + 4 + 5 = 10$

c $\log_a\left(\frac{\sqrt{x}}{y}\right) = \log_a\left(\sqrt{x}\right) - \log_a(y) = \frac{1}{2}\log_a(x) - \log_a(y) = \frac{1}{2} \times 4 - 5 = -3$

11 Show that $\log_3\left(\sqrt[4]{\frac{x^2}{y^8z^6}}\right) = \frac{1}{2}\log_3(x) - 2\log_3(y) + \frac{3}{2}\log_3(z)$

$$\begin{aligned}
\log_3\left(\sqrt[4]{\frac{x^2}{y^8z^6}}\right) &= \log_3\left(\frac{x^2}{y^8z^6}\right)^{\frac{1}{4}} \\
&= \frac{1}{4}\log_3\left(\frac{x^2}{y^8z^6}\right) \\
&= \frac{1}{4}\left[\log_3(x^2) - \log_3(y^8) - \log_3(z^6)\right] \\
&= \frac{2}{4}\log_3(x) - \frac{8}{4}\log_3(y) - \frac{6}{4}\log_3(z) \\
&= \frac{1}{2}\log_3(x) - 2\log_3(y) - \frac{3}{2}\log_3(z)
\end{aligned}$$

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12 **a** Prove: $\log_a(1) = 0$ for $a > 0$ and $a \neq 1$

Let $\log_a(1) = x$

Using the definition of logarithms,

$1 = a^x$

$\therefore\ x = 0$ for any $a > 0$. We do not use 1 as a base so $a \neq 1$.

b Prove: $\log_a(a) = 1$ for $a > 0$ and $a \neq 1$.

Let $\log_a(a) = x$

Using the definition of logarithms,

$a = a^x$

$\therefore\ x = 1$ for $a > 0$ and $a \neq 1$.

13 Prove the quotient law of logarithms

i.e. prove $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$ for $a, x, y > 0$ and $a \neq 1$.

Let $\log_a(x) = p$ and let $\log_a(y) = q$

$\therefore\ x = a^p$ and $y = a^q$

$$\frac{x}{y} = \frac{a^p}{a^q}$$

$$\frac{x}{y} = a^{p-q}$$

$$\therefore \log_a\left(\frac{x}{y}\right) = p - q$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y) \quad \text{for } a, x, y > 0 \text{ and } a \neq 1.$$

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Exercise 7.03 Common logarithms and the change of base theorem

Concepts and techniques

1 **a** $\log_4(9) = \dfrac{\log_{10}(9)}{\log_{10}(4)}$

b $\log_8(6) = \dfrac{\log_{10}(6)}{\log_{10}(8)}$

c $\log_2(20) = \dfrac{\log_{10}(20)}{\log_{10}(2)} = \dfrac{\log_{10}(2)+\log_{10}(10)}{\log_{10}(2)} = \dfrac{\log_{10}(2)}{\log_{10}(2)} + \dfrac{\log_{10}(10)}{\log_{10}(2)} = 1 + \dfrac{1}{\log_{10}(2)}$

d $\log_7(200) = \dfrac{\log_{10}(200)}{\log_{10}(7)} = \dfrac{\log_{10}(2)+\log_{10}(100)}{\log_{10}(7)} = \dfrac{\log_{10}(2)+2}{\log_{10}(7)}$

e $\log_9(0.2) = \dfrac{\log_{10}(0.2)}{\log_{10}(9)} = \dfrac{\log_{10}(2)-\log_{10}(10)}{\log_{10}(9)} = \dfrac{\log_{10}(2)-1}{\log_{10}(9)}$

2 **a** $\log_3(6) = \dfrac{\log_{10}(6)}{\log_{10}(3)} = \dfrac{0.7782}{0.4771} = 1.631$

b $\log_{12}(2) = \dfrac{\log_{10}(2)}{\log_{10}(12)} = \dfrac{0.3010}{1.0792} = 0.2789$

c $\log_5(15) = \dfrac{\log_{10}(15)}{\log_{10}(5)} = 1.6826$

d $\log_{25}(4) = \dfrac{\log_{10}(4)}{\log_{10}(25)} = 0.4307$

e $\log_8(1.3) = \dfrac{\log_{10}(1.3)}{\log_{10}(8)} = 0.1262$

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3 **a** $2^x = 100$

$$x = \log_2(100)$$
$$= \frac{\log_{10}(100)}{\log_{10}(2)}$$
$$x = 6.644$$

b $4^x = 9$

$$x = \log_4(9)$$
$$= \frac{\log_{10}(9)}{\log_{10}(4)}$$
$$x = 1.585$$

c $5^x = 70$

$$x = \log_5(70)$$
$$= \frac{\log_{10}(70)}{\log_{10}(5)}$$
$$x = 2.640$$

d $(0.75)^x = 0.01.$

$$x = \log_{0.75}(0.01)$$
$$= \frac{\log_{10}(0.01)}{\log_{10}(0.75)}$$
$$x = 16.01$$

e $(1.045)^x = 2$

$$x = \log_{1.045}(2)$$
$$= \frac{\log_{10}(2)}{\log_{10}(1.045)}$$
$$x = 15.75$$

4 **a** $3^x = 5$

$$x = \log_3(5)$$
$$= \frac{\log_{10}(5)}{\log_{10}(3)}$$
$$x = 1.465$$

b $7^x = 14.3$

$$x = \log_7(14.3)$$
$$= \frac{\log_{10}(14.3)}{\log_{10}(7)}$$
$$x = 1.367$$

c $3^x = 15$

$$x = \log_3(15)$$
$$= \frac{\log_{10}(15)}{\log_{10}(3)}$$
$$x = 2.465$$

d $5^x = 100$

$$x = \log_5(100)$$
$$= \frac{\log_{10}(100)}{\log_{10}(5)}$$
$$x = 2.861$$

e $6^x = 4$

$$x = \log_6(4)$$
$$= \frac{\log_{10}(4)}{\log_{10}(6)}$$
$$x = 0.774$$

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5 **a** $3^{x+1} = 85.7$

$$x+1=\log_3(85.7)$$
$$x+1=\frac{\log_{10}(85.7)}{\log_{10}(3)}$$
$$x+1=4.051$$
$$x=3.051$$

b $9^{4x+1} = 64$

$$4x+1=\log_9(64)$$
$$4x+1=\frac{\log_{10}(64)}{\log_{10}(9)}$$
$$4x+1=1.892\,789$$
$$4x=0.892\,789$$
$$x=0.2232$$

c $5^{2x+1} = 32$

$$2x+1=\log_5(32)$$
$$2x+1=\frac{\log_{10}(32)}{\log_{10}(5)}$$
$$2x+1=2.1533828$$
$$x=0.5767$$

d $3^{7x-2} = 13$

$$7x-2=\log_3(13)$$
$$7x-2=\frac{\log_{10}(13)}{\log_{10}(3)}$$
$$7x-2=4.051$$
$$x=0.6192$$

e $6^{5-3x} = 17$

$$5-3x=\log_6(17)$$
$$5-3x=\frac{\log_{10}(17)}{\log_{10}(6)}$$
$$5-3x=1.581\,246$$
$$x=1.140$$

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Reasoning and communication

6 Given $\log_3(a) = b$ and $\log_a(2) = c$, find $\log_a(48)$

$\log_a(48) = \log_a(16 \times 3)$

$= 4\log_a(2) + \log_a(3)$

$\log_3(a) = \dfrac{\log_a(a)}{\log_a(3)} = \dfrac{1}{\log_a(3)}$

$\therefore \quad \log_a(3) = \dfrac{1}{b}$

$\log_a(48) = 4\log_a(2) + \log_a(3)$

$= 4c + \dfrac{1}{b}$

7 Prove that $\log_a(b) = \dfrac{1}{\log_b(a)}$

$\log_a(b) = \dfrac{\log_b(b)}{\log_b(a)} = \dfrac{1}{\log_b(a)}$

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8 Given that $3^x = 4^y = 12^z$, show that $z = \dfrac{xy}{x+y}$

$\log_a\left(3^x\right) = \log_a\left(4^y\right) = \log_a\left(12^z\right)$ for any defined base a

$x\log_a(3) = y\log_a(4) = z\log_a(12)$

$\therefore\ x = \dfrac{z\log_a(12)}{\log_a(3)}$ and $y = \dfrac{z\log_a(12)}{\log_a(4)}$

$$xy = \frac{z\log_a(12)}{\log_a(3)} \times \frac{z\log_a(12)}{\log_a(4)}$$

$$x + y = \frac{z\log_a(12)}{\log_a(3)} + \frac{z\log_a(12)}{\log_a(4)}$$

$$= \frac{z\log_a(12)\log_a(4) + z\log_a(12)\log_a(3)}{\log_a(3)\log_a(4)}$$

$$= \frac{z\log_a(12)\left[\log_a(4) + \log_a(3)\right]}{\log_a(3)\log_a(4)}$$

$$= \frac{z\log_a(12)\log_a(12)}{\log_a(3)\log_a(4)}$$

$$\frac{xy}{x+y} = \frac{z\log_a(12)}{\log_a(3)} \times \frac{z\log_a(12)}{\log_a(4)} \div \frac{z\log_a(12)\log_a(12)}{\log_a(3)\log_a(4)}$$

$$\frac{xy}{x+y} = \frac{z\log_a(12)}{\log_a(3)} \times \frac{z\log_a(12)}{\log_a(4)} \times \frac{\log_a(3)\log_a(4)}{z\log_a(12)\log_a(12)}$$

$$\frac{xy}{x+y} = z$$

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Exercise 7.04 Solving equations with logarithms

Concepts and techniques

1 **a** $9^x + 3^x = 12$

$(3^x)^2 + 3^x = 12$

Let $p = 3^x$

$p^2 + p - 12 = 0$

$(p+4)(p-3) = 0$

$p = -4$ or $p = 3$

$\therefore\ 3^x = -4$ or $3^x = 3$

$3^x = -4$ has no solution as $3^x > 0$

$x = 1$ only

b $5^{2x+1} + 5^x - 4 = 0$

$5^1(5^x)^2 + 5^x - 4 = 0$

Let $p = 5^x$

$5p^2 + p - 4 = 0$

$(5p-4)(p+1) = 0$

$p = 0.8$ or $p = -1$

$\therefore\ 5^x = 0.8$ or $5^x = -1$

$5^x = -1$ has no solution as $5^x > 0$

$5^x = 0.8$ only

$x = \log_5(0.8)$

$$x = \frac{\log_{10}(0.8)}{\log_{10}(5)}$$

$x = -0.1386$

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c $1 + 6^{1-x} = 6^x$

$1 + 6^1\left(6^x\right)^{-1} = 6^x$

Let $p = 6^x$

$1 + \frac{6}{p} = p$

Multiply both sides by *p*.

$p + 6 = p^2$

$p^2 - p - 6 = 0$

$(p-3)(p+2) = 0$

$p = 3$ or $p = -2$

$\therefore$ but $6^x > 0$

$6^x = 3$ only

$x = \log_6(3)$

$x = \frac{\log_{10}(3)}{\log_{10}(6)}$

$x = 0.6131$

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d $2^{2x+1} + 20 = 3 \times 2^x$

$\left(2^x\right)^2 2 + 20 = 3\left(2^x\right)$

Let $p = 2^x$

$2p^2 - 3p + 20 = 0$

$(2p+5)(p-4) = 0$

$p = -2.5$ or $p = 4$

But $2^x > 0$

$2^x = 4$ only

$x = 2$

e $11 \times 8^x - 30 = 8^{2x}$

$11\left(8^x\right) - 30 = \left(8^x\right)^2$

Let $p = 8^x$

$p^2 - 11p + 30 = 0$

$(p-5)(p-6) = 0$

$p = 5$ or $p = 6$

$\therefore$ $8^x = 5$ or $8^x = 6$

$x = \log_8(5)$ or $x = \log_8(6)$

$x = \dfrac{\log_{10}(5)}{\log_{10}(8)}$ or $x = \dfrac{\log_{10}(6)}{\log_{10}(8)}$

$x = 0.774$ or $x = 0.862$

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2 **a** $9^x = 5^{x+3}$

$$x\log_{10}(9) = (x+3)\log_{10}(5)$$

$$x\left[\log_{10}(9) - \log_{10}(5)\right] = 3\log_{10}(5)$$

$$x = \frac{3\log_{10}(5)}{\left[\log_{10}(9) - \log_{10}(5)\right]}$$

b $8^x = 49^{x-3}$

$$x\log_{10}(8) = (x-3)\log_{10}(49)$$

$$x\left[\log_{10}(8) - \log_{10}(49)\right] = -3\log_{10}(49)$$

$$x = \frac{-3\log_{10}(49)}{\left[\log_{10}(8) - \log_{10}(49)\right]}$$

c $4^{x+5} = 350^{x-5}$

$$(x+5)\log_{10}(4) = (x-5)\log_{10}(350)$$

$$x\left[\log_{10}(4) - \log_{10}(350)\right] = -5\log_{10}(350) - 5\log_{10}(4)$$

$$x = \frac{-5\left[\log_{10}(350) + \log_{10}(4)\right]}{\left[\log_{10}(4) - \log_{10}(350)\right]}$$

d $2^{3x} = 15^{x-1}$

$$3x\log_{10}(2) = (x-1)\log_{10}(15)$$

$$x\left[3\log_{10}(2) - \log_{10}(15)\right] = -\log_{10}(15)$$

$$x = \frac{-\log_{10}(15)}{\left[3\log_{10}(2) - \log_{10}(15)\right]}$$

e $7^{2x-1} = 17^{x+2}$

$$(2x-1)\log_{10}(7) = (x+2)\log_{10}(17)$$

$$x\left[2\log_{10}(7) - \log_{10}(17)\right] = 2\log_{10}(17) + \log_{10}(7)$$

$$x = \frac{2\log_{10}(17) + \log_{10}(7)}{\left[2\log_{10}(7) - \log_{10}(17)\right]}$$

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3 **a** $4^x = 7^{x-2}$

$$x\log_{10}(4) = (x-2)\log_{10}(7)$$
$$x\left[\log_{10}(4) - \log_{10}(7)\right] = -2\log_{10}(7)$$
$$x = \frac{-2\log_{10}(7)}{\left[\log_{10}(4) - \log_{10}(7)\right]}$$
$$x = 6.954$$

b $58^x = 4^{x+4}$

$$x\log_{10}(58) = (x+4)\log_{10}(4)$$
$$x\left[\log_{10}(58) - \log_{10}(4)\right] = 4\log_{10}(4)$$
$$x = \frac{4\log_{10}(4)}{\left[\log_{10}(58) - \log_{10}(4)\right]}$$
$$x = 2.074$$

c $5^{x+2} = 46^{x-2}$

$$(x+2)\log_{10}(5) = (x-2)\log_{10}(46)$$
$$x\left[\log_{10}(5) - \log_{10}(46)\right] = -2\log_{10}(46) - 2\log_{10}(5)$$
$$x = \frac{-2\log_{10}(46) - 2\log_{10}(5)}{\left[\log_{10}(5) - \log_{10}(46)\right]}$$
$$x = 4.901$$

d $6^{2x} = 5^{x+3}$

$$2x\log_{10}(6) = (x+3)\log_{10}(5)$$
$$x\left[2\log_{10}(6) - \log_{10}(5)\right] = 3\log_{10}(5)$$
$$x = \frac{3\log_{10}(5)}{\left[2\log_{10}(6) - \log_{10}(5)\right]}$$
$$x = 2.446$$

e $28^{x+1} = 9^{2x-4}$

$$(x+1)\log_{10}(28) = (2x-4)\log_{10}(9)$$
$$x\left[\log_{10}(28) - 2\log_{10}(9)\right] = -4\log_{10}(9) - \log_{10}(28)$$
$$x = \frac{-4\log_{10}(9) - \log_{10}(28)}{\log_{10}(28) - 2\log_{10}(9)}$$
$$x = 11.41$$

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4 **a** $\log_3 (x - 2) = 4$

$x - 2 = 3^4$

$x = 83$

b $\log (2x - 10) = 2$

$2x - 10 = 10^2 = 100$

$2x = 110$

$x = 55$

c $\log_2 (2x + 12) - \log_2 (x) = \log_2 (4)$

$\log_2\left(\dfrac{2x+12}{x}\right) = \log_2(4)$

$\therefore \dfrac{2x+12}{x} = 4$

$2x + 12 = 4x$

$2x = 12$

$x = 6$

d $\log_2 (2x + 1) - \log_2 (x - 1) = \log_2 (4x - 4) + 2$

$\log_2 (2x + 1) - \log_2 (x - 1) = \log_2 [4(x - 1)] + \log_2 (4)$

$\log_2 (2x + 1) - \log_2 (x - 1) = \log_2 (4) + \log_2(x - 1) + \log_2 (4)$

$\log_2\left[\dfrac{2x+1}{(x-1)^2}\right] = \log_2(16)$

$\dfrac{2x+1}{(x-1)^2} = 16$

$2x + 1 = 16(x^2 - 2x + 1)$

$16x^2 - 34x + 15 = 0$

$(8x - 5)(2x - 3) = 0$

$x = \dfrac{5}{8}$ or $x = 1\dfrac{1}{2}$

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e log (3*x* + 6) – log (*x* + 2) = log (*x* – 2)

$$\log\left(\frac{3x+6}{x+2}\right)=\log(x-2)$$

$$\therefore\ \frac{3x+6}{x+2}=(x-2)$$

$$3x+6=(x-2)(x+2)$$

$$3x+6=x^2-4$$

$$x^2-3x-10=0$$

$$(x-5)(x+2)=0$$

$$x=5 \text{ or } x=-2 \text{ but } x\neq -2$$

$$x=5$$

f $\log_3(2x-4)-\log_3(x-1)=\log_3(x-2)$

$$\log_3\left(\frac{2x-4}{x-1}\right)=\log_3(x-2)$$

$$\therefore\ \frac{2x-4}{x-1}=(x-2)$$

$$2x-4=(x-2)(x-1)$$

$$2x-4=x^2-3x+2$$

$$x^2-5x+6=0$$

$$(x-3)(x-2)=0$$

$$x=3 \text{ or } x=2 \text{ but } x\neq 2$$

$$x=3$$

5 **a** $[\log(x)]^2-2\log(x)-3=0$

Let $y=\log_{10}(x)$

$$y^2-2y-3=0$$

$$(y-3)(y+1)=0$$

$$y=3 \text{ or } y=-1$$

But $y=\log_{10}(x)$

$$\therefore \log_{10}(x)=3 \qquad \log_{10}(x)=-1$$

$$x=10^3 \qquad x=10^{-1}$$

$$x=1000 \qquad x=0.1$$

b $[\log_2(x)]^2 - 2\log_2(x) = 8$

Let $y = \log_2(x)$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4$ or $y = -2$

But $y = \log_2(x)$

$\therefore \log_2(x) = 4 \qquad \log_2(x) = -2$

$x = 2^4 \qquad x = 2^{-2}$

$x = 16 \qquad x = 0.25$

c $[\log_2(x)]^2 + \log_2(x) - 2 = 0$

$y^2 + y - 2 = 0$

$(y-1)(y+2) = 0$

$y = 1$ or $y = -2$

But $y = \log_2(x)$

$\therefore \log_2(x) = 1 \qquad \log_2(x) = -2$

$x = 2^1 \qquad x = 2^{-2}$

$x = 2 \qquad x = 0.25$

d $[\log_5(x)]^2 = \log_5(x) + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2$ or $y = -1$

But $y = \log_5(x)$

$\therefore \log_5(x) = 2 \qquad \log_5(x) = -1$

$x = 5^2 \qquad x = 5^{-1}$

$x = 25 \qquad x = 0.2$

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e $[\log_3(x)]^2 - \log_3(x^4) + 3 = 0$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 3$ or $y = 1$

But $y = \log_3(x)$

$\therefore \log_3(x) = 3 \qquad \log_3(x) = -1$

$x = 3^3 \qquad x = 3^{-1}$

$x = 27 \qquad x = \frac{1}{3}$

f $[\log_5(x)]^2 - \log_5(x^5) - 24 = 0$

$y^2 - 5y - 24 = 0$

$(y-8)(y+3) = 0$

$y = 8$ or $y = -3$

But $y = \log_5(x)$

$\therefore \log_5(x) = 8 \qquad \log_5(x) = -3$

$x = 5^8 \qquad x = 5^{-3}$

$x = 390\,625 \qquad x = 0.008$

6 **a** $3[\log(x)]^2 + 5\log(x) - 4 = 0$

$3y^2 + 5y - 4 = 0$

Using the calculator to solve

$y = -2.257\,334$ or $y = 0.590\,667$

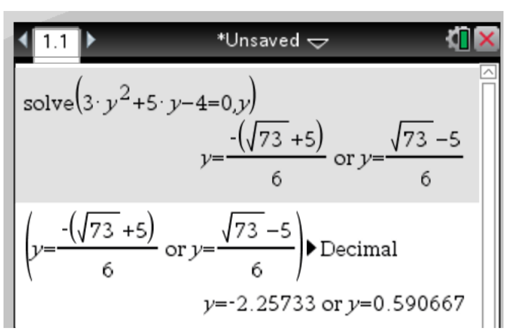
But $y = \log_{10}(x)$

$\therefore \log_{10}(x) = -2.257\,334 \qquad \log_{10}(x) = 0.590\,667$

$x = 10^{-2.257334} \qquad x = 10^{0.590667}$

$x = 0.005\,53 \qquad x = 3.90$

TI-Nspire CAS



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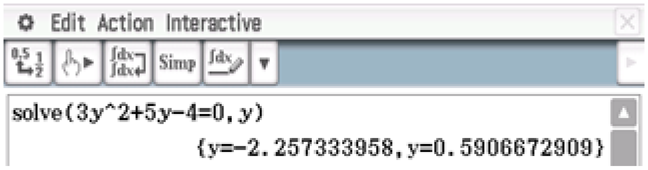


b $[\log_2(x)]^2 = 5\log_2(x) - 3$

$y^2 - 5y + 3 = 0$

Using the calculator to solve

$y = 0.697\,224$ or $y = 4.302\,776$

But $y = \log_2(x)$

$\therefore \log_2(x) = 0.697\,224$ $\quad$ $\log_2(x) = 4.302\,776$

$x = 2^{0.697\,224}$ $\quad$ $x = 2^{4.302\,776}$

$x = 1.62$ $\quad$ $x = 19.7$

c $4[\log_3(x)]^2 = 6 - \log_3(x)$

$4y^2 + y - 6 = 0$

Using the calculator to solve

$y = -1.356187$ or $y = 1.106107$

But $y = \log_3(x)$

$\therefore \log_3(x) = -1.356187$ $\quad$ $\log_3(x) = 1.106107$

$x = 3^{-1.356187}$ $\quad$ $x = 3^{1.106107}$

$x = 0.225$ $\quad$ $x = 3.37$

d $2\log(x) - 4 = 3[\log(x)]^2$

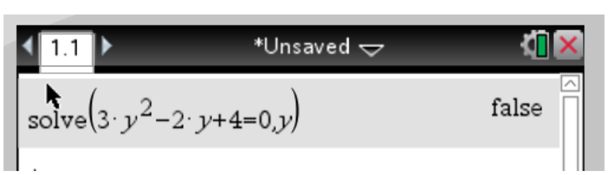
$2y - 4 = 3y^2$

$3y^2 - 2y + 4 = 0$

Using the calculator to solve.

There are no real solutions.

TI-Nspire CAS



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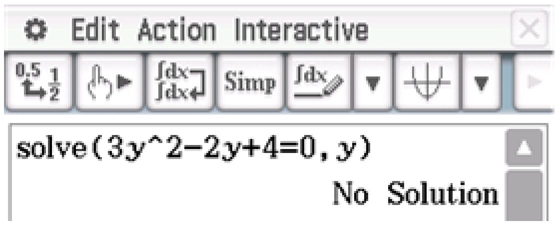

e $[\log_5(x)]^2 + 5\log_5(x) - 3 = 0$

$y^2 + 5y - 3 = 0$

Using the calculator to solve.

$y = -5.541\ldots$ or $y = 0.541\ldots$

But $y = \log_5(x)$

So $x = 5^{-5.541\ldots} \approx 0.00134$ or $x = 5^{0.541\ldots} \approx 2.39$

Reasoning and communication

7 $A = P\left(1+\frac{i}{k}\right)^{kn}$

$$5000 = 1000\left(1+\frac{i}{2}\right)^{2\times 20}$$

$$5 = \left(1+\frac{i}{2}\right)^{40}$$

$$\log_{10}(5) = 40\log_{10}\left(1+\frac{i}{2}\right)$$

$$\frac{\log_{10}(5)}{40} = \log_{10}\left(1+\frac{i}{2}\right)$$

$$\left(1+\frac{i}{2}\right) = 10^{\frac{\log_{10}(5)}{40}}$$

$$i = 2\times\left(10^{\frac{\log_{10}(5)}{40}} - 1\right)$$

$i = 0.0821$ (i.e. 8.21%)

8 Formula is: $A = P\left(1+\frac{i}{k}\right)^{kn}$

Money doubles itself, so $A = 2P$; assume yearly compound interest, so $k = 1$; and $n = 15$.

$$2P = P\left(1+i\right)^{15k}$$

$$2 = \left(1+i\right)^{15}$$

$$2^{\frac{1}{15}} = 1+i$$

$$i = 2^{\frac{1}{15}} - 1$$

$$8P = P\left(1+i\right)^{t}$$

$$8 = \left(1+2^{\frac{1}{15}}-1\right)^{t}$$

$$8 = 2^{\frac{1}{15}}$$

$$\log_{10}(8) = \frac{t}{15}\log_{10}(2)$$

$$t = 15\times\frac{\log_{10}(8)}{\log_{10}(2)}$$

$$t = 45 \text{ years}$$

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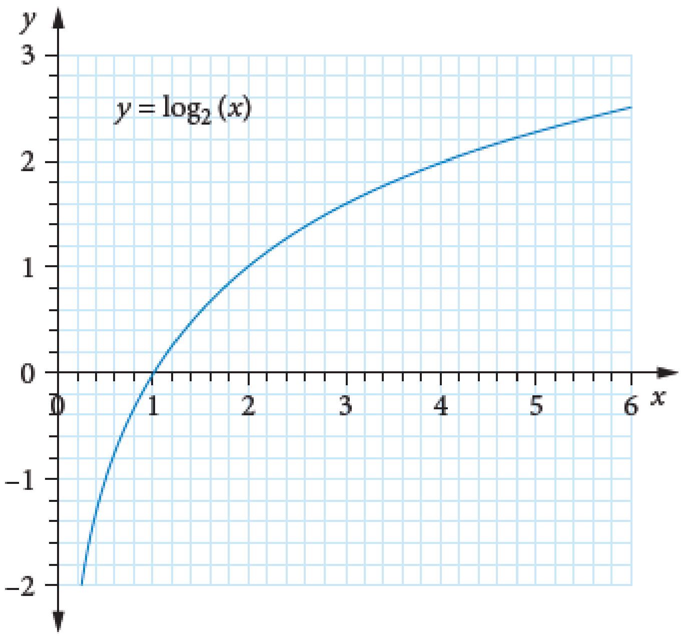
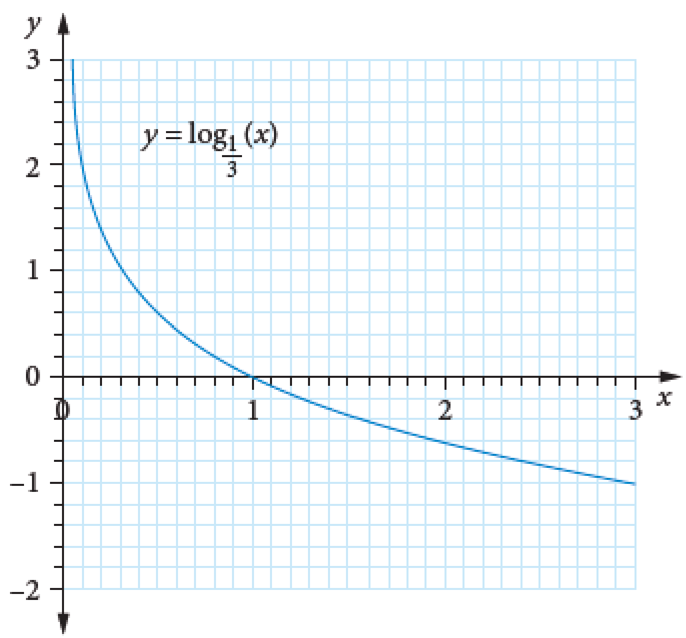
Exercise 7.05 Logarithmic graphs

Concepts and techniques

1 **a** $\log_2(x) = 2.3$, $x \approx 4.9$



b $\log_{\frac{1}{3}}(y) = 1.4$, $y \approx 0.2$



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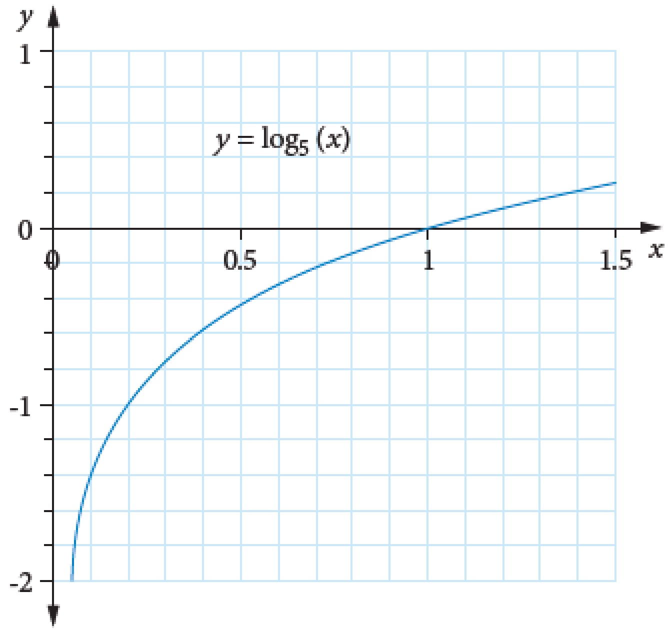
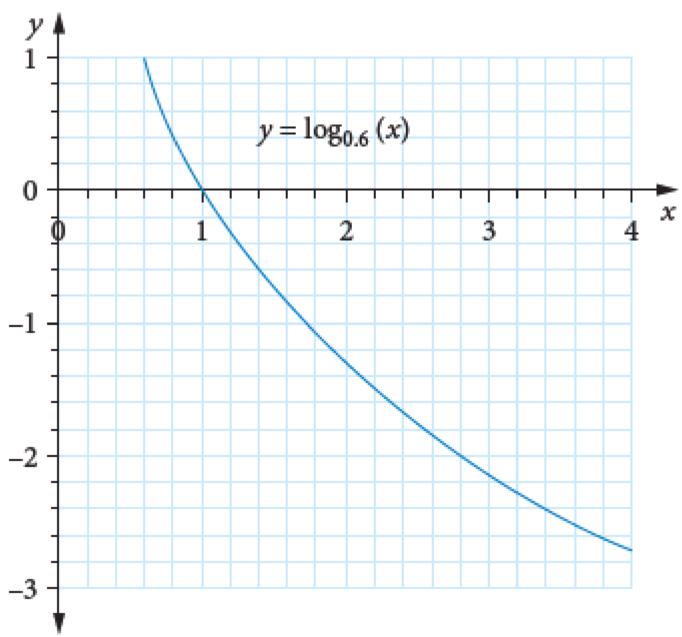
c $\log_5(z) = -0.8$, $z \approx 0.27$



d $\log_{0.6}(k) = -1.7$, $k \approx 2.4$



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2 **a** $\log_4(x) = 3.1$, $x \approx 75$

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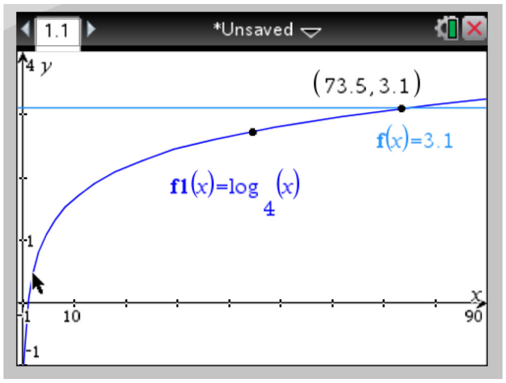

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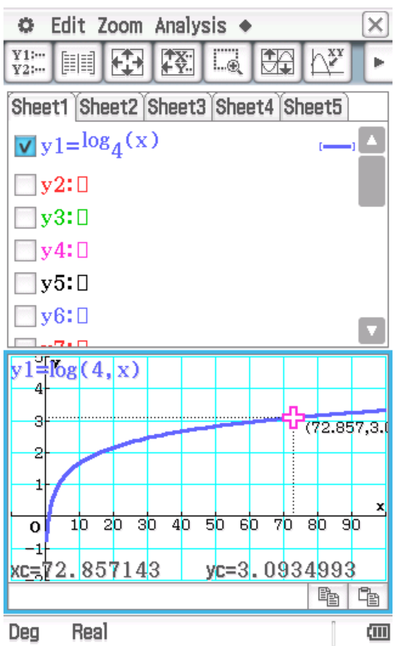
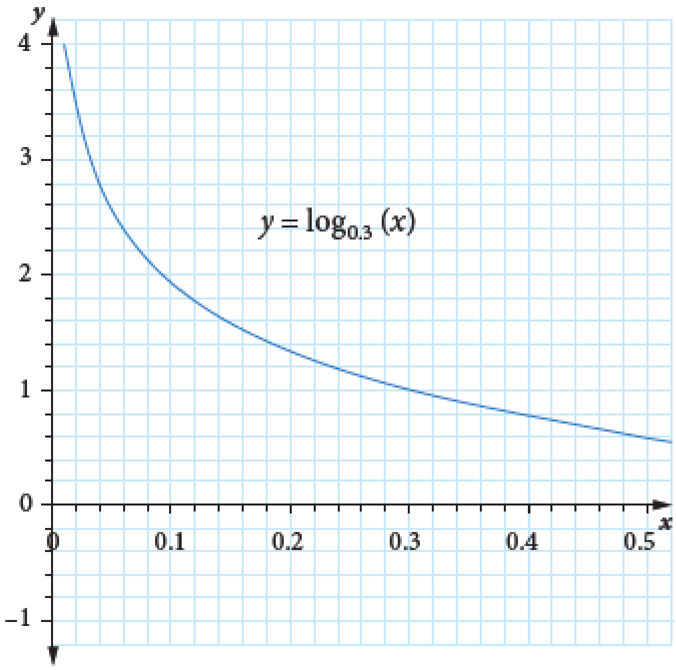


b $\log_{0.3}(y) = 2.5$, $y \approx 0.05$



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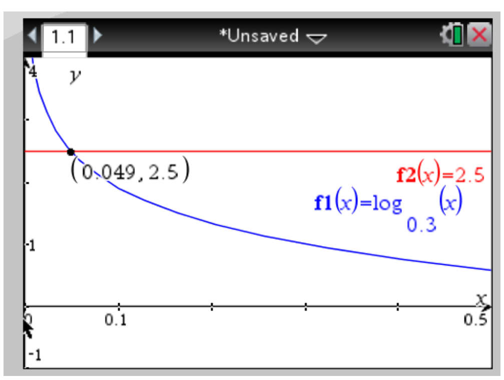

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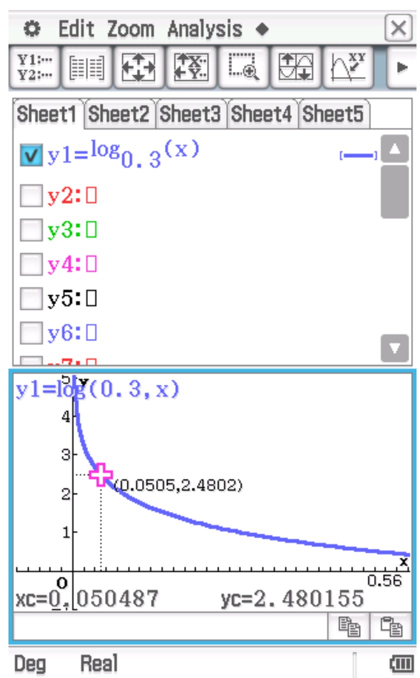

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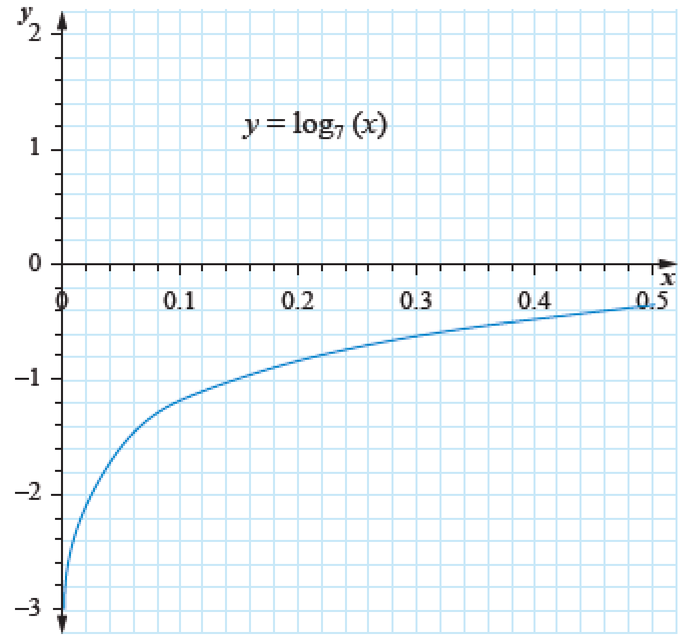
c $\log_7(z) = -1.6,\ z = 7^{-3.6} \approx 0.04$



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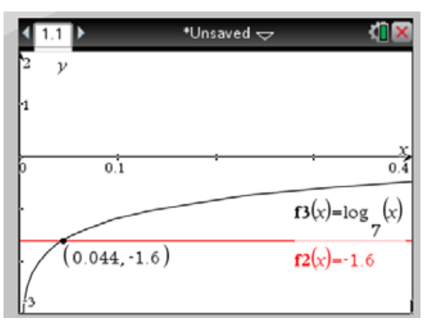

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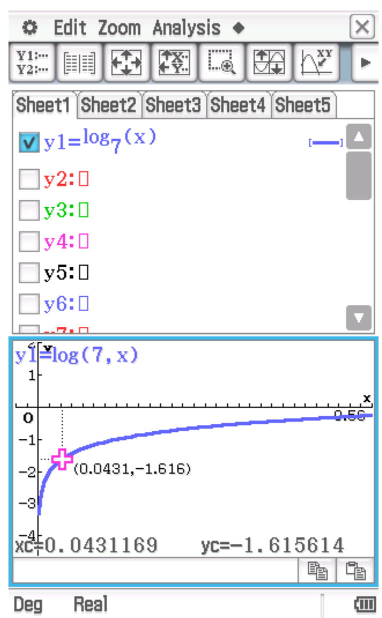

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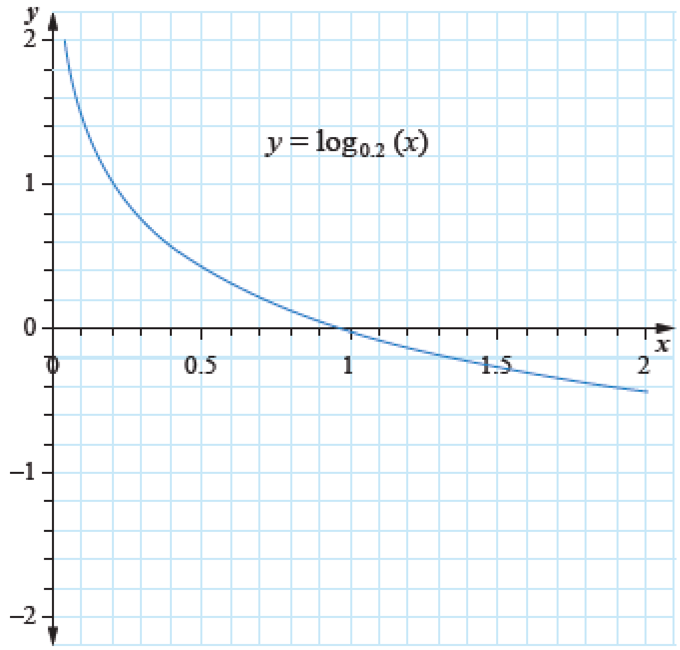
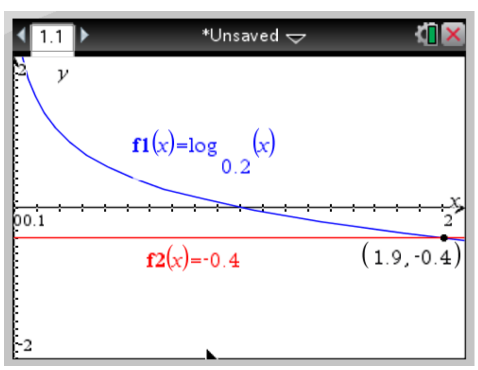
d $\log_{0.2}(k) = -0.4$, $k \approx 1.9$



TI-Nspire CAS



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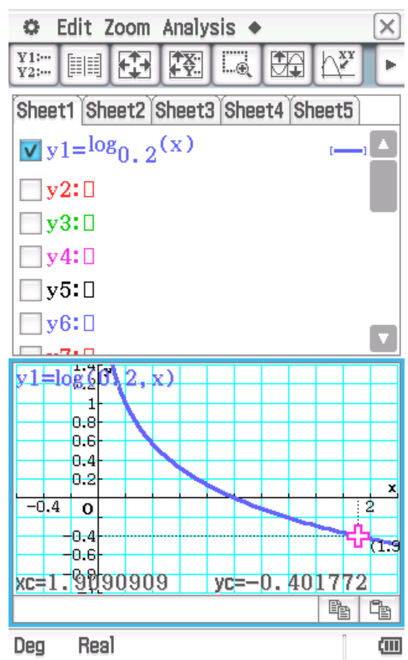

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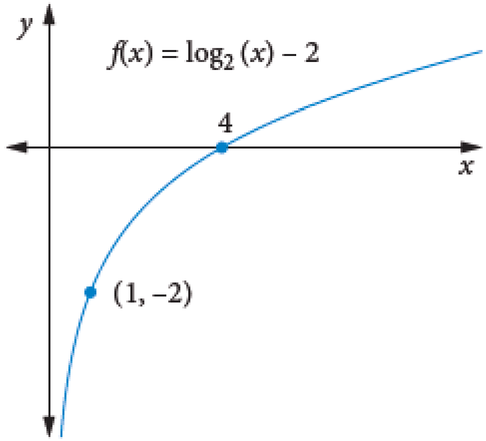
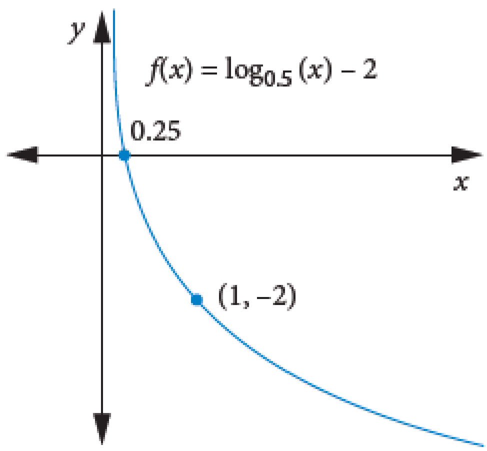
3 **a** $f(x) = \log_2(x) - 2$

Vertical translation 2 down from $f(x) = \log_2(x)$



b $f(x) = \log_{0.5}(x) - 2$

Vertical translation 2 down from $f(x) = \log_{0.5}(x)$



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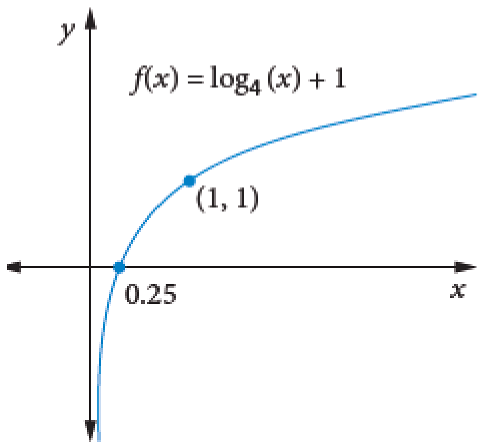
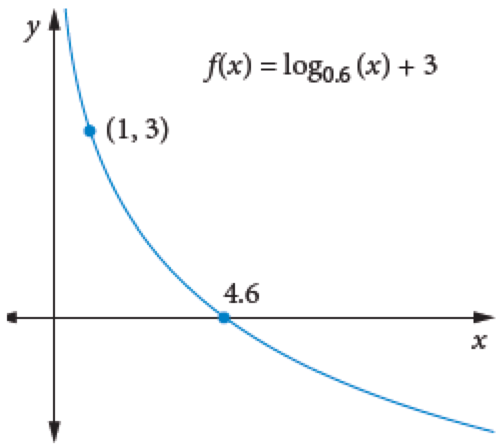
c $f(x) = \log_4(x) + 1$

Vertical translation 1 up from $f(x) = \log_4(x)$



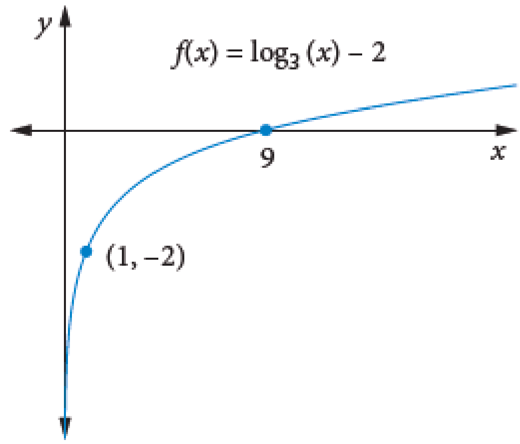
d $f(x) = \log_{0.6}(x) + 3$

Vertical translation 3 up from $f(x) = \log_{0.6}(x)$



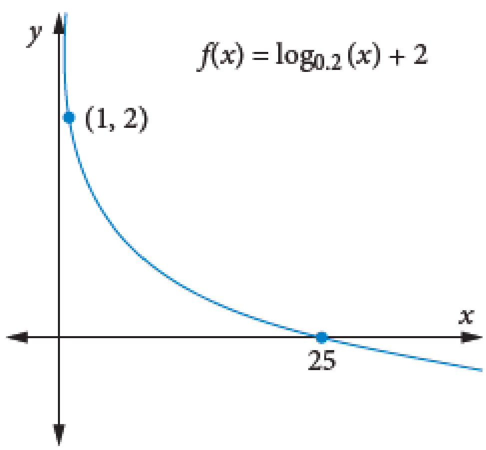
e $f(x) = \log_3(x) - 2$

Vertical translation 2 down from $f(x) = \log_3(x)$



f $f(x) = \log_{0.2}(x) + 2$

Vertical translation 2 up from $f(x) = \log_{0.2}(x)$



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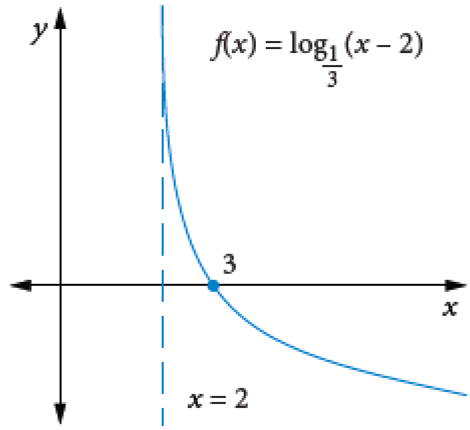
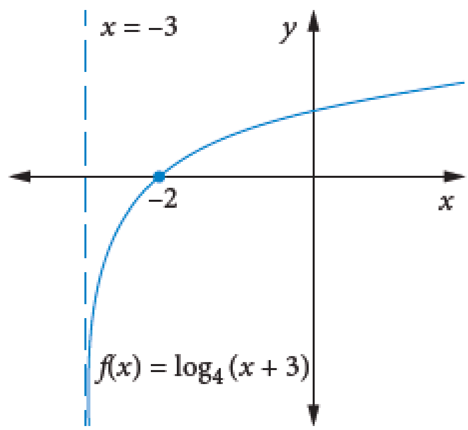
4 **a** $f(x) = \log_{\frac{1}{3}}(x-2)$

Horizontal translation 2 right from $f(x) = \log_{\frac{1}{3}}(x)$



b $f(x) = \log_4(x+3)$

Horizontal translation 3 left from $f(x) = \log_4(x)$



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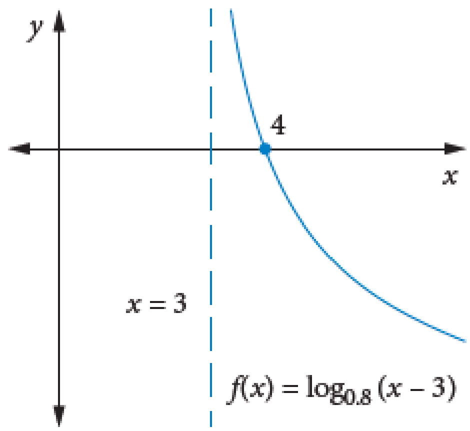
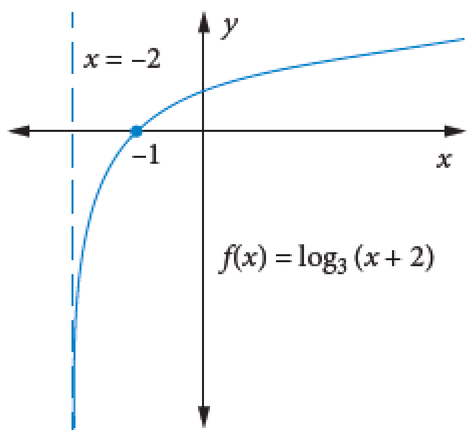
c $f(x) = \log_{0.8}(x - 3)$

Horizontal translation 3 right from $f(x) = \log_{0.8}(x)$



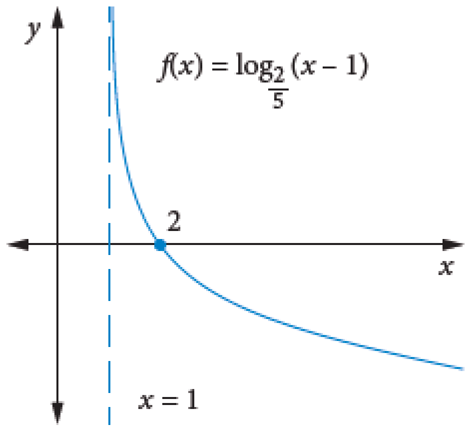
d $f(x) = \log_3(x + 2)$

Horizontal translation 2 left from $f(x) = \log_3(x)$



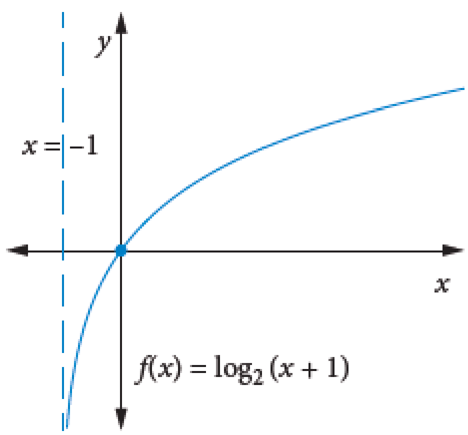
e $f(x) = \log_{\frac{2}{5}}(x-1)$

Horizontal translation 1 right from $f(x) = \log_{\frac{2}{5}}(x)$



f $f(x) = \log_2(x+1)$

Horizontal translation 1 left from $f(x) = \log_2(x)$



5 **a** $f(x) = \log_7(x) + 3$

b $f(x) = \log_{0.5}(x) - 2$

c $f(x) = \log_{\frac{1}{6}}(x) + 1$

d $f(x) = \log_3(x) - 4$

6 **a** $f(x) = \log_5(x + 4)$

b $f(x) = \log_{0.3}(x - 2)$

c $f(x) = \log_4(x + 3)$

d $f(x) = \log_{0.8}(x - 5)$

Reasoning and communication

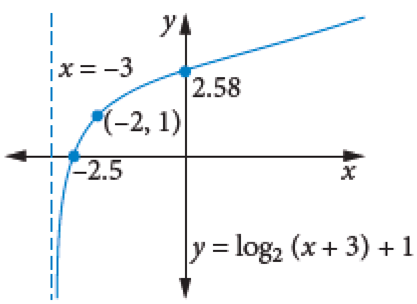
7 **a** $f(x) = \log_2(x + 4) + 3$

b $f(x) = \log_{0.1}(x - 2) + 1$

c $f(x) = \log_4(x + 3) - 4$

d $f(x) = \log_{0.6}(x - 1) - 2$

8 **a** $y = \log_2(x + 3) + 1$



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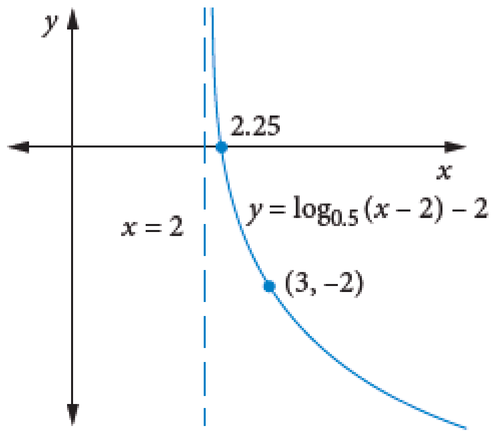
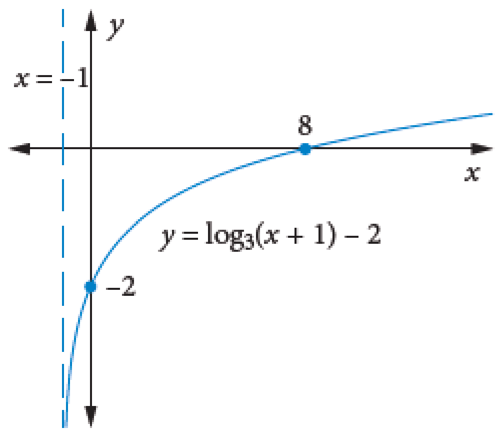
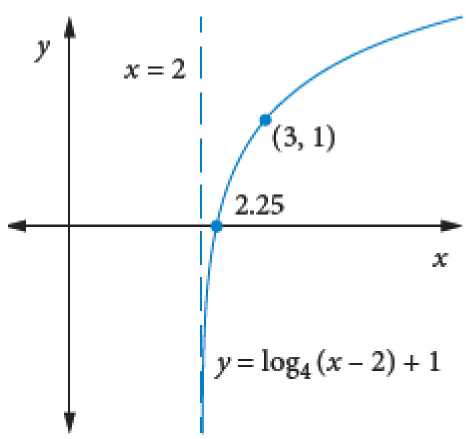
b $y = \log_{0.5}(x - 2) - 2$



c $y = \log_3(x + 1) - 2$



d $y = \log_4(x - 2) + 1$



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Exercise 7.06 Applications of logarithms

Reasoning and communication

1 $s = 930 \log(d) + 65$

$280 = 930\log(d) + 65$

$215 = 930\log(d)$

$\log(d) = \frac{215}{930}$

$d = 10^{\frac{215}{930}}$

$= 1.7029$

The tornado has travelled about 1700 km over warm ocean water.

2 $pH = -\log([H^+])$

a $[H^+] \times [OH^-] = 10^{-14}$ and $[OH^-] = 10^{-1}$

$\therefore [H^+] = 10^{-13}$

$pH = -\log(10^{-13}) = 13$

b $pH = 15$

$10^{-15} \times [OH^-] = 10^{-14}$

$\therefore [OH^-] = 10$

The concentration of OH^- ions is 10 moles/litre.

3 $pH = -2$

$pH = -\log([H^+])$

$-2 = -\log([H^+])$

$[H^+] = 10^2$

The concentration of hydrogen ions is 100 moles/litre.

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4 $n = k\log(A)$

When $A = 500\text{ km}^2$, $n = 2800$

$2800 = k\log(500)$

$k = 1037.43$

$n = ?$ when $A = 250$

$n = 1037.43\log(250)$

$= 2487.78$

≈ 2488

5 $S = 10\log\left(\frac{I}{I_0}\right)$

$125 = 10\log\left(\frac{I}{1\times10^{-12}}\right)$

$12.5 = \log\left(\frac{I}{1\times10^{-12}}\right)$

$\frac{I}{1\times10^{-12}} = 10^{12.5}$

$I = 10^{12.5}\times1\times10^{-12}$

$= 10^{0.5} = 3.16$

6 $L = 9 + 5.1\log(d)$

a $L = 9 + 5.1\log(6)$

$L = 12.9686$

b $10 = 9 + 5.1\log(d)$

$\frac{1}{5.1} = \log(d)$

$d = 10^{\frac{1}{5.1}}$

$d = 1.5706$

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7 $R = 0.67 \log(0.37E) + 1.46$

a $R = 0.67 \log(0.37 \times 15\,500\,000\,000) + 1.46$

≈ 7.998

≈ 8

b $8.5 = 0.67 \log(0.37E) + 1.46$

$$\frac{7.04}{0.67} = \log(0.37E)$$

$$0.37E = 10^{\frac{7.04}{0.67}} = 3.217\ 088\ 6117 \times 10^{10}$$

$$E = 8.695 \times 10^{10}$$

8 $y = a + b \log_2(t)$

At $t = 1$, $y = 18.27$

$18.27 = a + b \log_2(1)$, but $\log_2(1) = 0$

$\therefore a = 18.27$

At $t = 2$, $y = 25.41$

$25.41 = 18.27 + b \log_2(2)$, but $\log_2(2) = 1$

$7.14 = b$

$\therefore y = 18.27 + 7.14 \log_2(t)$

At $t = 10$, $y = ?$

$y_{10} = 18.27 + 7.14 \log_2(10)$

$y_{10} = 41.989$ litres

After 10 hours from a desalination process, there are 42 litres of fresh water produced.

9 **a** Each note has frequency given by, say, k times the frequency of the previous note. There are 12 notes in the scale, so the next note of the same name has frequency k^{12} times the frequency of the previous one of the same name. But this is double, so $k^{12} = 2$ and $k = \sqrt[12]{2}$.

b There are 3 steps from A to C through A# and B, so middle C has frequency $(\sqrt[12]{2})^3 \times 440 = \sqrt[4]{2} \times 440$ Hz

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Exercise 7.07 The natural logarithm and its derivative

Concepts and techniques

1 **a** $y = \ln(x)$

$$\frac{dy}{dx} = \frac{1}{x}$$

b $y = \ln(10x)$

$$\frac{dy}{dx} = \frac{10}{10x} = \frac{1}{x}$$

c $y = 3\ln(2x)$

$$\frac{dy}{dx} = 3 \times \frac{2}{2x} = \frac{3}{x}$$

d $y = \ln(0.3x)$

$$\frac{dy}{dx} = \frac{0.3}{0.3x} = \frac{1}{x}$$

e $y = 6\ln(9x)$

$$\frac{dy}{dx} = 6 \times \frac{9}{9x} = \frac{6}{x}$$

f $y = \ln\left(\frac{x}{2}\right)$

$$\frac{dy}{dx} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x}$$

g $y = 4\ln\left(\frac{x}{3}\right)$

$$\frac{dy}{dx} = 4 \times \frac{\frac{1}{3}}{\frac{x}{3}} = \frac{4}{x}$$

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h $y = 2\ \ln\left(-\frac{2x}{3}\right)$

$$\frac{dy}{dx} = 2 \times \frac{-\frac{2}{3}}{\frac{2x}{3}} = -\frac{2}{x}$$

2 **a** $\frac{d}{dx}\log_4(x) = \frac{1}{x\ln(4)}$

b $\frac{d}{dx}\log(x) = \frac{1}{x\ln(10)}$

c $\frac{d}{dx}\log_9(x) = \frac{1}{x\ln(9)}$

d $\frac{d}{dx}\log_2(x) = \frac{1}{x\ln(2)}$

e $\frac{d}{dx}\log_{0.2}(x) = \frac{1}{x\ln(0.2)} = \frac{1}{x\ln\left(\frac{1}{5}\right)} = \frac{1}{x\ln(5^{-1})} = -\frac{1}{x\ln(5)}$

3 **a** $y = \ln(3x - 1)$

$$\frac{dy}{dx} = \frac{3}{3x-1}$$

b $y = \ln(2x + 7)$

$$\frac{dy}{dx} = \frac{2}{2x+7}$$

c $y = 2\ \ln(4x - 3)$

$$\frac{dy}{dx} = 2 \times \frac{4}{4x-3} = \frac{8}{4x-3}$$

d $y = 5\ \ln(6x + 7)$

$$\frac{dy}{dx} = 5 \times \frac{6}{6x+7} = \frac{30}{6x+7}$$

e $y = \ln(2x + 1)$

$$\frac{dy}{dx} = \frac{2}{2x+1}$$

f $y = 3 \ln (5x - 1)$

$$\frac{dy}{dx} = 3 \times \frac{5}{5x-1} = \frac{15}{5x-1}$$

g $y = 6 \ln (3 - 4x)$

$$\frac{dy}{dx} = 6 \times \frac{-4}{3-4x} = \frac{-24}{3-4x} = \frac{24}{4x-3}$$

h $y = 12 \ln (5 - 8x)$

$$\frac{dy}{dx} = 12 \times \frac{-8}{5-8x} = \frac{-96}{5-8x} = \frac{96}{8x-5}$$

4 **a** $y = \ln (3x^5)$

$$\frac{dy}{dx} = \frac{15x^4}{3x^5} = \frac{5}{x}$$

b $y = \ln (4x^3)$

$$\frac{dy}{dx} = \frac{12x^2}{4x^3} = \frac{3}{x}$$

c $y = \ln (2x^2 + 1)$

$$\frac{dy}{dx} = \frac{4x}{2x^2+1}$$

d $y = 2 \ln (5 - 8x^2)$

$$\frac{dy}{dx} = 2 \times \frac{-16x}{(5-8x^2)} = \frac{32x}{8x^2-5}$$

e $y = \ln (x^3 - 2x^2 + 3x - 4)$

$$\frac{dy}{dx} = \frac{3x^2-4x+3}{x^3-2x^2+3x-4}$$

f $y = 3 \ln (2x^4 - 7x^5 + x)$

$$\frac{dy}{dx} = 3 \times \frac{8x^3-35x^4+1}{(2x^4-7x^5+x)} = \frac{3\left(8x^3-35x^4+1\right)}{2x^4-7x^5+x}$$

5 **a** $y = \ln (\sqrt{3x+1})$

$$y = \frac{1}{2}\ln(3x+1)$$

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{3}{(3x+1)} = \frac{3}{2(3x+1)}$$

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b $y = \ln\left(\sqrt{5-7x}\right)$

$$y = \frac{1}{2}\ln(5-7x)$$

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{-7}{(5-7x)} = \frac{7}{2(7x-5)}$$

c $y = \ln\left(\sqrt[3]{4x+9}\right)$

$$y = \frac{1}{3}\ln(4x+9)$$

$$\frac{dy}{dx} = \frac{1}{3} \times \frac{4}{(4x+9)} = \frac{4}{3(4x+9)}$$

d $y = \ln\left(\sqrt[5]{8-x}\right)$

$$y = \frac{1}{5}\ln(8-x)$$

$$\frac{dy}{dx} = \frac{1}{5} \times \frac{-1}{(8-x)} = -\frac{1}{5(8-x)}$$

e $y = \ln(3x-7)^4$

$$y = 4\ln(3x-7)$$

$$\frac{dy}{dx} = 4 \times \frac{3}{(3x-7)} = \frac{12}{3x-7}$$

f $y = \ln(5x-2)^3$

$$y = 3\ln(5x-2)$$

$$\frac{dy}{dx} = 3 \times \frac{5}{(5x-2)} = \frac{15}{5x-2}$$

g $y = \ln\left(\frac{1}{x+2}\right)$

$$y = \ln(x+2)^{-1}$$

$$y = -\ln(x+2)$$

$$\frac{dy}{dx} = -1 \times \frac{1}{(x+2)} = -\frac{1}{x+2}$$

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h $y=\ln\left(\frac{2}{5-3x}\right)$

$$y=\ln(2)-\ln(5-3x)$$

$$\frac{dy}{dx}=0+(-1)\times\frac{-3}{(5-3x)}=\frac{3}{5-3x}$$

i $y=\ln\left(\frac{3}{6x+1}\right)^{-2}$

$$y=-2\ln\left(\frac{3}{6x+1}\right)$$

$$y=-2\ln(3)+2\ln(6x+1)$$

$$\frac{dy}{dx}=0+2\times\frac{6}{(6x+1)}=\frac{12}{6x+1}$$

j $y=\ln\left(\frac{7}{4-x}\right)^{-5}$

$$y=-5\ln(7)+5\ln(4-x)$$

$$\frac{dy}{dx}=0+5\times\frac{-1}{(4-x)}=\frac{5}{x-4}$$

6 **a** $y=\ln(x^2+2)^2$

$$y=2\ln(x^2+2)$$

$$\frac{dy}{dx}=2\times\frac{2x}{(x^2+2)}=\frac{4x}{x^2+2}$$

b $y=\ln(3-x^2)^2$

$$y=2\ln(3-x^2)$$

$$\frac{dy}{dx}=2\times\frac{-2x}{(3-x^2)}=\frac{-4x}{3-x^2}$$

c $y=\ln(x^3-2x+3)^3$

$$y=3\ln(x^3-2x+3)$$

$$\frac{dy}{dx}=3\times\frac{3x^2-2}{(x^3-2x+3)}=\frac{3\left(3x^2-2\right)}{x^3-2x+3}$$

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d $y = \ln (2x^3 - 3x^2 + 4x - 1)^3$

$$y = 3\ln(2x^3 - 3x^2 + 4x - 1)$$

$$\frac{dy}{dx} = 3 \times \frac{6x^2 - 6x + 4}{(2x^3 - 3x^2 + 4x - 1)} = \frac{6\left(3x^2 - 3x + 2\right)}{2x^3 - 3x^2 + 4x - 1}$$

7 **a** $\frac{d}{dx}(x^2 - 2x + 1)\ln(x)$

$$= \left(2x - 2\right)\ln(x) + \frac{1}{x}(x^2 - 2x + 1)$$

b $\frac{d}{dx}\ (x^3 + 3x^2 + 5)\ln(x^3 + 3x^2 + 5)$

$$= \left(3x^2 + 6x\right) \times \ln\left(x^3 + 3x^2 + 5\right) + \frac{\left(3x^2 + 6x\right)}{\left(x^3 + 3x^2 + 5\right)} \times \left(x^3 + 3x^2 + 5\right)$$

$$= \left(3x^2 + 6x\right) \times \ln\left(x^3 + 3x^2 + 5\right) + \frac{\left(3x^2 + 6x\right)}{\cancel{\left(x^3 + 3x^2 + 5\right)}} \times \cancel{\left(x^3 + 3x^2 + 5\right)}$$

$$= \left(3x^2 + 6x\right)\left[\ln\left(x^3 + 3x^2 + 5\right) + 1\right]$$

$$= 3x\left(x + 2\right)\left[\ln\left(x^3 + 3x^2 + 5\right) + 1\right]$$

c $\frac{d}{dx} x\ln(x)$

$$= 1 \times \ln(x) + \frac{1}{x} \times x$$

$$= 1 + \ln(x)$$

d $\frac{d}{dx} e^x \ln(x)$

$$= e^x \times \ln(x) + \frac{1}{x} \times e^x$$

$$= e^x\left[\ln(x) + \frac{1}{x}\right]$$

e $\frac{d}{dx}\ln(x)\sin(x)$

$$= \frac{1}{x}\times\sin(x) + \cos(x)\times\ln(x)$$
$$= \frac{\sin(x)}{x} + \cos(x)\times\ln(x)$$

f $\frac{d}{dx}\left\{\ln(x)\cos(x) + \frac{\sin(x)}{x}\right\}$

$$= \frac{1}{x}\times\cos(x) - \sin(x)\times\ln(x) + \frac{x\cos(x) - 1\times\sin(x)}{x^2}$$
$$= \frac{\cos(x)}{x} - \sin(x)\times\ln(x) + \frac{\cos(x)}{x} - \frac{\sin(x)}{x^2}$$
$$= \frac{2\cos(x)}{x} - \sin(x)\ln(x) - \frac{\sin(x)}{x^2}$$

Reasoning and communication

8 Given $f(x) = 6\ln(3 - 4x)$

a $f'(x) = 6\times\frac{-4}{(3-4x)}$

$$f'(x) = \frac{-24}{(3-4x)}$$

b $f'(2) = \frac{-24}{-5} = \frac{24}{5}$

c $f'(x) = 2$, $x = ?$

$$2 = \frac{-24}{(3-4x)}$$
$$(3-4x) = -12$$
$$15 = 4x$$
$$x = 3.75$$

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9 Given $f(x) = 6\ln\left(\sqrt{x^2-1}\right) = 6\times\frac{1}{2}\ln(x^2-1) = 3\ln(x^2-1)$

a $f'(x) = 3\times\frac{2x}{x^2-1} = \frac{6x}{x^2-1}$

b $f'(2) = \frac{6\times 2}{4-1} = 4$

c $f'(x) = 6$, $x = ?$

$$\frac{6x}{x^2-1} = 6$$
$$x^2-1 = x$$
$$x^2-x-1 = 0$$
$$x = \frac{1\pm\sqrt{5}}{2}$$

But for $x = \frac{1-\sqrt{5}}{2}$, $f(x)$ is not defined, so $x = \frac{1+\sqrt{5}}{2}$.

10 Given $f(x) = 4x^2 + 3\ln(x^2+2x)$

a $f'(x) = 8x + 3\times\frac{2x+2}{\left(x^2+2x\right)}$

$$= 8x + \frac{6(x+1)}{\left(x^2+2x\right)}$$

b $f'(2) = 16 + 2.25 = 18.25$

c $f'(x) = 2$

$$8x + \frac{6(x+1)}{\left(x^2+2x\right)} = 2$$
$$4x + \frac{3(x+1)}{\left(x^2+2x\right)} = 1$$
$$4x\left(x^2+2x\right) + 3(x+1) = \left(x^2+2x\right)$$
$$4x^3 + 7x^2 + x + 3 = 0$$
$$x = -1.836$$

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11 Given $g(x) = \ln\left[f(x)\right]$, $f(1) = 3$ and $f'(1) = 6$, $g'(1) = ?$

$$g'(x) = \frac{f'(x)}{f(x)}$$

$$\therefore g'(1) = \frac{6}{3}$$

$$g'(1) = 2$$

12 Prove that $\frac{d}{dx}\{\ln[f(x)]\} = \frac{f'(x)}{f(x)}$.

Let $y = \ln[f(x)]$

$$\frac{dy}{dx} = \frac{1}{f(x)} \times f'(x)$$

$$\therefore \frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

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Exercise 7.08 The integral of $\frac{1}{x}$

Concepts and techniques

1 **a** $\int \frac{2}{x}\,dx = 2\ln(x) + c$

b $\int \frac{7}{x}\,dx = 7\ln(x) + c$

c $\int \frac{6}{5x}\,dx = \frac{6\ln(x)}{5} + c$

d $\frac{4}{7}\int \frac{1}{x}\,dx = \frac{4\ln(x)}{7} + c$

e $-\frac{8}{11}\int \frac{1}{x}\,dx = -\frac{8\ln(x)}{11} + c$

f $-\frac{9}{4}\int \frac{1}{x}\,dx = -\frac{9\ln(x)}{4} + c$

2 **a** $\int \frac{1}{x+4}\,dx = \ln(x+4) + c$ for $x > -4$

b $\int \frac{1}{x-2}\,dx = \ln(x-2) + c$ for $x > 2$

c $\int \frac{1}{3x+1}\,dx = \frac{\ln(3x+1)}{3} + c$ for $x > -\frac{1}{3}$

d $\int \frac{1}{5x-9}\,dx = \frac{\ln(5x-9)}{5} + c$ for $x > \frac{9}{5}$

e $\int \frac{11}{7x-9}\,dx = \frac{11\ln(7x-9)}{7} + c$ for $x > \frac{9}{7}$

f $\int \frac{13}{4x-1}\,dx = \frac{13\ln(4x-1)}{4} + c$ for $x > \frac{1}{4}$

g $\int \frac{6}{5-2x}\,dx = -6\int \frac{1}{2x-5}\,dx = \frac{-6\ln(2x-5)}{2} + c = -3\ln(2x-5) + c$ for $x > 2.5$

h $\int \frac{7}{3-x}\,dx = \frac{-7\ln(x-3)}{1} + c = -7\ln(x-3) + c$ for $x > 3$

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Reasoning and communication

3 **a** $\int \frac{x^3+x^2}{x^3}dx = \int 1+\frac{1}{x}\,dx = x+\ln(x) \quad \text{for } x>0$

b $\int \frac{4x^4-3x^2+x}{x^3}dx = \int\left(4x-\frac{3}{x}+x^{-2}\right)dx$

$$= 2x^2-3\ln(x)+\frac{x^{-1}}{-1}+c$$

$$= 2x^2-3\ln(x)-\frac{1}{x}+c \quad \text{for } x>0$$

c $\int \frac{5x+2x^3-1}{x^2}dx = \int \frac{5}{x}+2x-x^{-2}\,dx$

$$= 5\ln(x)+x^2+\frac{1}{x}+c \text{ for } x>0$$

d $\int \frac{4x^2+8x^5-2x}{2x^3}dx = \int \frac{2}{x}+4x^2-x^{-2}\,dx$

$$= 2\ln(x)+\frac{4x^3}{3}+\frac{1}{x}+c \quad \text{for } x>0$$

e $\int \frac{3x^{10}-2x^4+15x^2}{x^3}dx = \int 3x^7-2x+\frac{15}{x}dx$

$$= \frac{3x^8}{8}-x^2+15\ln(x)+c \quad \text{for } x>0$$

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4 $f'(x) = \dfrac{1}{x-2}$

$$f(x) = \int \frac{dx}{x-2}$$
$$f(x) = \ln(x-2) + c$$
$$f(3) = 6$$
$$6 = \ln(1) + c$$
$$f(x) = \ln(x-2) + 6 \quad \text{for } x > 2$$

5 $f'(x) = \dfrac{7}{5-3x}$

$$f(x) = \int \frac{7}{5-3x}\,dx = -7\int \frac{7}{3x-5}\,dx = \frac{-7\ln(3x-5)}{3} + c$$
$$f(2) = 7$$
$$7 = -\frac{7\ln(1)}{3} + c, \quad c = 7$$
$$f(x) = -\frac{7}{3}\ln(3x-5), \; x > \frac{5}{3}$$

6 $\dfrac{d}{dx}[\ln(x^2+2)] = \dfrac{2x}{(x^2+2)}$

$$\therefore \int \frac{4x}{x^2+2}\,dx = 2\int \frac{2x}{x^2+2}\,dx = 2\ln(x^2+2) + c\,.$$

7 $\dfrac{d}{dx}[\ln(x^2-5)] = \dfrac{2x}{(x^2-5)}$

$$\therefore \int \frac{x}{x^2-5}\,dx = \frac{1}{2}\int \frac{2x}{x^2-5}\,dx = \frac{1}{2}\ln(x^2-5) + c\,, \; x^2 > 5$$

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8 $$\frac{1}{x-2}-\frac{1}{x+2}=\frac{x+2-(x-2)}{(x-2)(x+2)}$$

$$=\frac{4}{(x-2)(x+2)}$$

$$=\frac{4}{x^2-4}$$

$$\therefore \int\frac{4}{x^2-4}\,dx=\int\frac{1}{x-2}-\frac{1}{x+2}\,dx$$

$$=\ln(x-2)-\ln(x+2)+c \quad \text{for } x>2$$

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Exercise 7.09 Applications of natural logarithms

Reasoning and communication

1 $L(t) = 100 \ln (kt)$

a $129 = 100 \ln (20k)$

$$20k = e^{1.29}$$

$$k = \frac{e^{1.29}}{20}$$

b $L(10) = 100 \ln (10k)$

$$L(10) = 100 \ln\left(\frac{e^{1.29}}{20} \times 10\right)$$

$$= 59.68$$

The student will not yet have learnt 60, so 59.

c $$L(t) = 100 \ln\left(\frac{e^{1.29}}{20} t\right)$$

$$L(60) = 100 \ln\left(\frac{e^{1.29}}{20} \times 60\right)$$

$$= 238.86$$

The student will have learnt 238 words.

d $180 = 100 \ln\left(\frac{e^{1.29}}{20} t\right)$, $t = 33.3$ minutes

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e $L(t) = 100\ \ln\left(\frac{e^{1.29}}{20}t\right)$

$$L'(t) = 100 \times \frac{\frac{e^{1.29}}{20}}{\left(\frac{e^{1.29}}{20}t\right)}$$

$$L'(t) = \frac{100}{t}$$

At $t = 45$ minutes,

$$L'(45) = \frac{100}{45}$$

$L'(45) = 2.3$ words per minute

2 $N(t) = 500 \ln(21t + 3),\ t \in [0, 40]$

a $N(0) = 500 \ln(3)$, on 1 Jan, $0 \le t < 1$

$N(0) = 549.3$

About 549 moths.

b $N(30) = 500 \ln(21 \times 30 + 3)$

$N(30) \approx 3225$

c $2000 = 500 \ln(21t + 3)$

$t = 2.457$

i.e. 3 January

d $P(t) = P \ln(Qt + 3),\ t \in [0, 40]$

At $t = 0$, $769 = P \ln(3) \Rightarrow P = \frac{769}{\ln(3)} = 699.97 \approx 700$

At $t = 15$, $2750 = 700 \ln(Q \times 15 + 3) \Rightarrow 15Q + 3 = e^{\frac{2750}{700}} \Rightarrow Q = 3.189$

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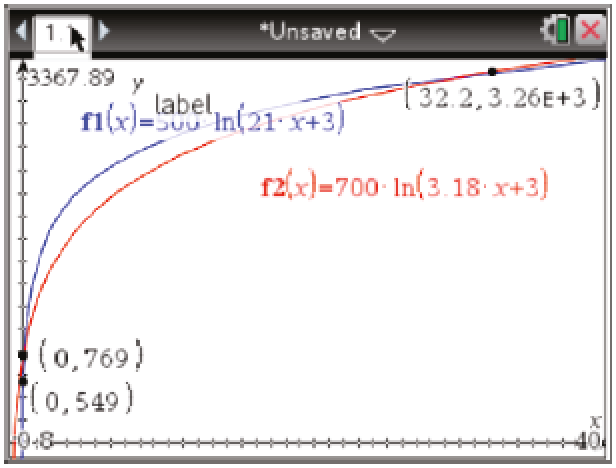
e



f $t = 32.2$, i.e. 2 February

g $N(t) = 500 \ln(21t + 3)$

$$N'(t) = \frac{500 \times 21}{(21t + 3)}$$

$$N'(32.2) = \frac{500 \times 21}{(21 \times 32.2 + 3)} = 15.6 \text{ moths/day}$$

$$P(t) = 700 \ln(3.189t + 3)$$

$$P'(t) = \frac{700 \times 3.189}{(3.189t + 3)}$$

$$P'(32.2) = 21.1 \text{moths/day}$$

3 **a** $60 = 60 - a \log(0 - b)$

$a \log(-b) = 0$, so $-b = 1$, i.e. $b = -1$

$55 = 60 - a \log(12 + 1)$

$a \log(13) = 5$, $a = 4.488\ldots$

Thus $T(t) = 60 - 4.488\ldots \times \log(t + 1)$

b $50 = 60 - a \log(t + 1)$

$\log(t + 1) = \dfrac{10}{a} = 2.227\ldots$

$t + 1 = 169$, so $t = 168$

c $T'(t) = \dfrac{-a}{t+1}$, so at $t = 168$, the rate of change is $\dfrac{-a}{169} = -0.0265\ldots$ s/day

The rate of change is a reduction of about 0.027 s/day.

d $46 = 60 - a \log(t + 1)$

$\log(t + 1) = \dfrac{14}{a} = 3.119\ldots$

$t + 1 = 1315.35\ldots$

$t = 1314.35\ldots$

To get *under* 46 seconds, it would take 1315 days of intensive training, or more than $3\frac{1}{2}$ years. This is probably impossible to maintain.

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4 $W(t) = W_0 - a \ln(t + 1)$

$185 = W_0 - a \ln(1) \Rightarrow 185 = W_0$

$W(t) = 185 - a \ln(t + 1)$

$W'(t) = -\frac{a}{(t+1)}$

$-0.2 = -\frac{a}{(30+1)}$

$a = 6.2$

$W(t) = W_0 - a \ln(t + 1)$

$100 = 185 - 6.2\ln(t+1)$

$-85 = -6.2\ln(t+1)$

$t+1 = e^{\frac{85}{6.2}}$

$t = 899573.7$ days

i.e. 2564 years... too long for him!

5 Assume the model is $N = N_0 + k \ln(t + 1)$

a At $t = 0$, $n = 5$, $5 = N_0 + k \ln(1)$

$N = 5 + k \ln(t + 1)$

At $t = 2$, $n = 7$

$7 = 5 + k \ln(3)$

$k = 1.82\ldots$

$N = 5 + 1.82\ldots \ln(t + 1)$

b $10 = 5 + 1.82\ldots \ln(t + 1)$

$1.82\ldots \ln(t + 1) = 5$

$t + 1 = 15.588\ldots$

$t = 14.588\ldots$

It will take 15 weeks to get up to at least 10 a day.

c $N = 5 + 1.82...\ln(t + 1)$

$$\frac{dN}{dt} = \frac{1.82...}{t+1}$$

At $t = 4$, $\frac{dN}{dt} = 0.364$ per week

d At $t = 10$, $\frac{dN}{dt} = 0.165$ per week

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Multiple choice

1 C $\log_2(8) = 3$ as $8 = 2^3$

2 A undefined, as there is no solution for $-2 = 3^x$

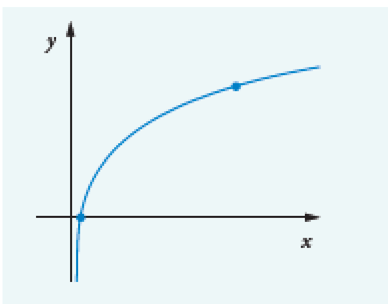
3 D $x > 5$ as you cannot obtain the log of 0 or a negative number.

$(x - 5) = 2^y$, $2^y > 0$, so $x - 5 > 0$, $x > 5$

4 B $\log_4(2) + \log_4(8) + \log_4\left(\frac{1}{4}\right) = \log_4\left(2\times 8\times \frac{1}{4}\right) = \log_4(4) = 1$

5 C $5\log(x) + 6\log(x + 6) = \log(x^5) + \log(x+6)^6 = \log\left[x^5(x+6)^6\right]$

6 E The graph of $y = \log_2(x) + 4$ is most like:



7 A $\frac{d}{dz}\ln(2x-3) = \frac{2}{(2x-3)}$

8 C $\int \frac{1}{5x}\,dx = \frac{1}{5}\ln(x)$

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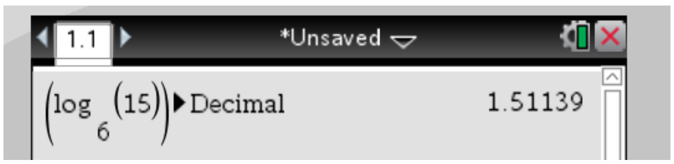
Short answer

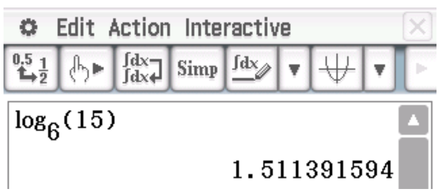
9 $\log_3(81) = 4$ in index form is $81 = 3^4$.

10 $5^{-2} = 0.04$ in logarithmic form is $\log_5(0.04) = -2$

11 TI-Nspire CAS

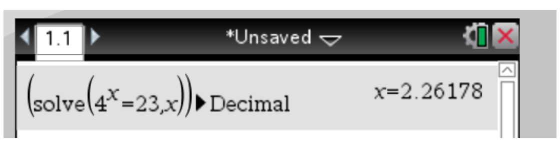


ClassPad

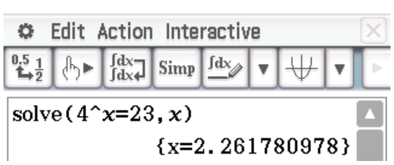


$$\log_6(15) = \frac{\log_{10}(15)}{\log_{10}(6)} = 1.511$$

12 TI-Nspire CAS



ClassPad



Solve $4^x = 23$

$x \log_{10}(4) = \log_{10}(23)$

$x = 2.262$

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13 Solve $3^{2x+1} + 3^x + 4 = 3^{x+2}$

Let $y = 3^x$

$3(y)^2 + y + 4 = 9y$

$3y^2 - 8y + 4 = 0$

$(3y - 2)(y - 2) = 0$

$y = \frac{2}{3}$ or $y = 2$

$3^x = \frac{2}{3}$ or $3^x = 2$

$x = -0.3690\ldots$ or $x = 0.6309\ldots$

14 Solve $4^{3x+2} = 6^{2x-1}$

$(3x + 2)\log_{10}(4) = (2x - 1)\log_{10}(6)$

$x[3\log_{10}(4) - 2\log_{10}(6)] = -\log_{10}(6) - 2\log_{10}(4)$

$$x = \frac{-\log_{10}(6) - 2\log_{10}(4)}{3\log_{10}(4) - 2\log_{10}(6)} = -7.933$$

15 Solve $2\log_3(x) + \log_3(2x - 1) - \log_3(x) = 1$

$$\log_3\left(\frac{x^2(2x-1)}{x}\right) = 1$$

$x(2x-1) = 3$

$2x^2 - x - 3 = 0$

$(2x-3)(x+1) = 0$

$x = 1.5$ or $x = -1$, but $x > 0$

$\therefore x = 1.5$

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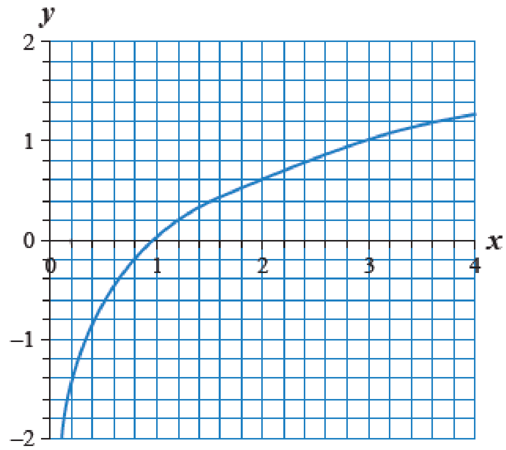
16 $\log_3(x) = -0.7$, $x \approx 0.46$



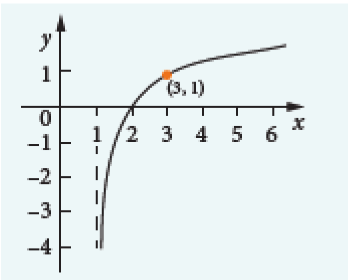
17 $y = \log_a(x + b)$

$x > 1$, so $x - 1 > 0$

$b = -1$

Now at $x = 3$, $y = 1$

Thus $\log_a(2) = 1$ so $a = 2$ and the function is $y = \log_2(x - 1)$.



18 **a** $$\frac{d}{dx}\log_6(x) = \frac{d}{dx}\frac{\ln(x)}{\ln(6)} = \frac{1}{x\ln(6)}$$

b $$\frac{d}{dx}\log_e\left(3x^2 + 8\right) = \frac{6x}{\left(3x^2 + 8\right)}$$

c $$\frac{d}{dx}\left[(3x^4 - x^3 + 5)\ln\left(7x + 1\right)\right] = \left(12x^3 - 3x^2\right)\ln\left(7x + 1\right) + \frac{7(3x^4 - x^3 + 5)}{\left(7x + 1\right)}$$

d $$\frac{d}{dx}\left[\frac{\ln(x)}{\ln(3x-5)}\right] = \frac{\dfrac{\ln(3x-5)}{x} - \dfrac{3\ln(x)}{(3x-5)}}{\left[\ln(3x-5)\right]^2}$$

$$= \frac{1}{\left[\ln(3x-5)\right]^2}\left[\frac{\ln(3x-5)}{x} - \frac{3\ln(x)}{(3x-5)}\right]$$

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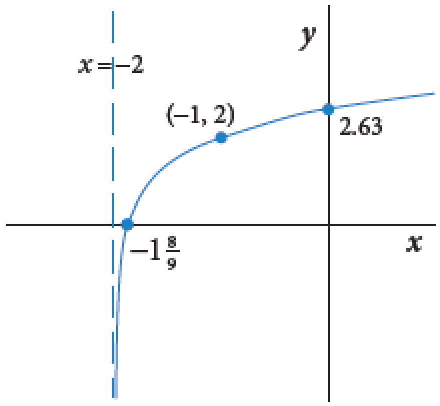
19 $\int \frac{3x^2 - 4}{x^3 - 4x + 1}\,dx = \ln(x^3 - 4x + 1) + c$

Application

20 $f(x) = \log_3(x + 2) + 2$



21 $SN = -\frac{7}{3}\log\left(\frac{T}{I}\right) + 1$

$10 = -\frac{7}{3}\log\left(\frac{T}{I}\right) + 1$

$\log\left(\frac{T}{I}\right) = 9 \times \frac{-3}{7}$

$\log\left(\frac{T}{I}\right) = \frac{-27}{7}$

$\frac{T}{I} = 10^{\frac{-27}{7}} = 0.000\,138...$

About 0.014%.

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22 $\int \frac{x^3 - 3x^2 + 2x - 4}{x+1} dx = ?$

Dividing:

$$\begin{array}{r|l} & x^2 - 4x + 6 \\ x+1 & x^3 - 3x^2 + 2x - 4 \\ & \underline{x^3 + x^2} \\ & \quad -4x^2 + 2x \\ & \quad \underline{-4x^2 - 4x} \\ & \qquad\qquad 6x - 4 \\ & \qquad\qquad \underline{6x + 6} \\ & \qquad\qquad\quad -10 \end{array}$$

$$\frac{x^3 - 3x^2 + 2x - 4}{x+1} = x^2 - 4x + 6 - \frac{10}{x+1}$$

$$\int \frac{x^3 - 3x^2 + 2x - 4}{x+1} dx = \int \left(x^2 - 4x + 6 - \frac{10}{x+1} \right) dx$$

$$= \frac{x^3}{3} - 2x^2 + 6x - 10\ln(x+1) + c$$

Alternative method

Using synthetic division

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 2 & -4 \\ & & -1 & 4 & -6 \\ \hline & 1 & -4 & 6 & -10 \end{array}$$

$$\frac{x^3 - 3x^2 + 2x - 4}{x+1} = x^2 - 4x + 6 - \frac{10}{x+1}$$

$$\int \frac{x^3 - 3x^2 + 2x - 4}{x+1} dx = \int \left(x^2 - 4x + 6 - \frac{10}{x+1} \right) dx$$

$$= \frac{x^3}{3} - 2x^2 + 6x - 10\ln(x+1) + c$$

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23 $x = \log_e(2t + e^2),\ t \geq 0$

a **i** At $t = 0$

$$x = \log_e(e^2)$$
$$x = 2\ln(e)$$
$$x = 2\text{ units}$$

ii 5 h =300 minutes

$$x = \log_e(600 + e^2)$$
$$x = 6.4\ \text{ units}$$

b $x = \log_e(2t + e^2),\ t \geq 0$

$$\frac{dx}{dt} = \frac{2}{(2t + e^2)}$$

At t = 180 minutes, $\dfrac{dx}{dt} = \dfrac{2}{(2 \times 180 + e^2)} = 0.00544\ldots$ units per minute

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